

# An NP Decision Procedure for Protocol Insecurity with XOR<sup>★</sup>

Yannick Chevalier<sup>a,1</sup>, Ralf Küsters<sup>b</sup>, Michaël Rusinowitch<sup>a</sup>,  
Mathieu Turuani<sup>a</sup>

<sup>a</sup>*LORIA-INRIA-Université Henri Poincaré, 54506 Vandoeuvre-les-Nancy, France*

<sup>b</sup>*Christian-Albrechts-Universität zu Kiel, Inst. f. Informatik, 24098 Kiel, Germany*

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## Abstract

We provide a method for deciding the insecurity of cryptographic protocols in presence of the standard Dolev-Yao intruder (with a finite number of sessions) extended with so-called oracle rules, i.e., deduction rules that satisfy certain conditions. As an instance of this general framework, we obtain that protocol insecurity is in NP for an intruder that can exploit the properties of the exclusive or (XOR) operator. This operator is frequently used in cryptographic protocols but cannot be handled in most protocol models. An immediate consequence of our proof is that checking whether a message can be derived by an intruder (using XOR) is in PTIME. We also apply our framework to an intruder that exploits properties of certain encryption modes such as cipher block chaining (CBC).

*Key words:* Verification, Security, XOR, Protocols, Complexity

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## 1 Introduction

Cryptographic protocols have been designed for handling secure electronic communications. Verification tools based on formal methods (e.g. model check-

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*Email addresses:* [ychevali@irit.fr](mailto:ychevali@irit.fr) (Yannick Chevalier),  
[kuesters@ti.informatik.uni-kiel.de](mailto:kuesters@ti.informatik.uni-kiel.de) (Ralf Küsters), [rusi@loria.fr](mailto:rusi@loria.fr) (Michaël Rusinowitch), [туруани@loria.fr](mailto:туруани@loria.fr) (Mathieu Turuani).

<sup>1</sup> Present address: IRIT, 118 route de Narbonne, 31062 Toulouse Cedex 04, France

ing) have been quite successful in discovering new flaws in published security protocols [18,23,29,9,3,4].

While most formal analysis of security protocols abstracts from algebraic properties of operators, such as the multiplicativity of RSA or the properties induced by chaining methods for block ciphers, many real attacks and protocol weaknesses rely on these properties. A typical example is an attack on Bull's recursive authentication protocol first observed by Ryan and Schneider [28]: The protocol is used to distribute a connected chain of keys linking all the nodes from the originator to the server. Ryan and Schneider show that if one key is compromised the others can be compromised too thanks to the property of the exclusive or (XOR). Conversely, if XOR is considered a free operator then, as shown by L. Paulson using the Isabelle prover [25], the protocol is secure.

Recently, several procedures have been proposed to decide insecurity of cryptographic protocols w.r.t. a finite number of protocol sessions [1,5,15,27,22,17]. Moreover, some special cases for an unbounded number of sessions have been studied [13,14,11,2] (see also [20,30] for related work). All these results assume encryption to be perfect (*perfect encryption assumption*): One needs a decryption key to extract the plaintext from the ciphertext, and also, a ciphertext can be generated only with the appropriate key and message (no collision). Only very few works on formal analysis have relaxed this assumption. In [21,16], unification algorithms are designed for handling properties of Diffie-Hellman cryptographic systems.

In this paper, we generalize the decidability result of [27], stating that insecurity for finitely many protocol sessions is in NP, to the case where messages may contain the XOR operator and where the Dolev-Yao intruder is extended by the ability to compose messages with the XOR operator. More precisely, we give a linear bound on the size of messages exchanged in minimal attacks and present an NP procedure for deciding insecurity with XOR. This extension is non-trivial due to the complex interaction of the XOR properties and the standard Dolev-Yao intruder rules. The technical problems raised by the equational laws are somewhat related to those encountered in semantic unification.

To prove our result, we have extended the Dolev-Yao intruder with so-called oracle rules, i.e., deduction rules that satisfy certain conditions. In this general framework we show that insecurity is decidable in NP. Now, the results for XOR are obtained by proving that the XOR rules satisfy the conditions on oracle rules.

Our framework is general enough to also handle other algebraic properties. More specifically, we show that the Dolev-Yao intruder equipped with the ability to exploit prefix properties of encryption algorithms based on cipher-

block-chaining (CBC) also falls into our framework.

To the best of our knowledge, the decidability results presented here (see also [6]) are the first, besides the ones by Comon and Shmatikov [12], that go beyond the perfect encryption assumption. We briefly compare our work with [12]: We prove that, in presence of the XOR operator, the problem of checking whether a message can be derived by an intruder is in PTIME—this problem is called *derivation problem* here and *ground reachability problem* in [12]. In [12], the derivation problem is shown to be in NP for the XOR operator and for abelian groups, respectively. As for the general insecurity problem, we show NP-completeness based on a theorem that ensures the existence of attacks of linear size. Comon and Shmatikov present a decision procedure with a higher complexity. This procedure is based on constraint solving techniques. However, they consider a more general class of protocol rules. In Section 3.2, we argue that these more general rules are rather unrealistic. Finally, we believe that our framework is quite general in the sense that different intruders with different deduction capabilities can be captured such as those for exploiting properties of encryption based on block ciphers (see Section 7).

*Structure of the paper.* In the following section, we provide an example illustrating the role of XOR in attacks. We then, in Section 3, introduce our protocol and intruder model. In particular, this section contains the definition of the oracle rules. The decidability result for the general framework is presented in Section 4, including the description of the NP decision algorithm. Proofs are provided in Section 5 and 6. Then, in Section 7, XOR rules and prefix rules are introduced and it is shown that these rules are oracle rules, which implies the mentioned complexity results. This section also contains an example illustrating the additional power of prefix rules in attacks.

## 2 A Motivating Example

We demonstrate that when taking the algebraic properties of XOR into account, new attacks can occur. As an example, we use a variant of the Needham-Schroeder-Lowe Protocol [19], i.e., the public-key Needham-Schroeder Protocol with Lowe’s fix, where in some place, instead of concatenation XOR is used. Using common notation (e.g.  $\{m\}_{K_X}^p$  denotes the encryption of message  $m$  with the public key of agent  $X$ ), the protocol is given as follows:

1.  $A \rightarrow B : \{N_A, A\}_{K_B}^p$
2.  $B \rightarrow A : \{N_B, \text{XOR}(N_A, B)\}_{K_A}^p$
3.  $A \rightarrow B : \{N_B\}_{K_B}^p$

If XOR is interpreted as free symbol, such as pairing, then according to [19] this protocol is secure. In particular, the intruder is not able to get hold of  $N_B$ . However, if the algebraic properties of XOR are taken into account, the following attack is possible, which is a variant of the original attack on the Needham-Schroeder Protocol and which allows the intruder  $I$  to obtain  $N_B$ . In this attack, two sessions run interleaved where the steps of the second session are marked with '. In the first session,  $A$  talks to the intruder  $I$ , and in the second session  $I$ , purporting to be  $A$ , talks to  $B$ .

1.  $A \rightarrow I$  :  $\{N_A, A\}_{K_I}^p$
- 1'.  $I(A) \rightarrow B$  :  $\{\text{XOR}(N_A, B, I), A\}_{K_B}^p$
- 2'.  $B \rightarrow I(A)$  :  $\{N_B, \text{XOR}(N_A, B, I, B)\}_{K_A}^p$
2.  $I \rightarrow A$  :  $\{N_B, \text{XOR}(N_A, B, I, B)\}_{K_A}^p$
3.  $A \rightarrow I$  :  $\{N_B\}_{K_I}^p$

In step 1'. of this attack,  $I$  first decrypts the message  $\{N_A, A\}_{K_I}^p$  to obtain  $N_A$  and  $A$ . Then  $I$  applies the XOR operator to compute  $\text{XOR}(N_A, B, I)$ , before this message together with  $A$  is encrypted for  $B$ . In 2'. and 2. it is used that the messages  $\text{XOR}(N_A, B, I, B)$  and  $\text{XOR}(N_A, I)$  are equivalent w.r.t. the properties of XOR, i.e.,  $\text{XOR}(N_A, B, I, B) =_{\text{XOR}} \text{XOR}(N_A, I)$ , and hence, the message in 2. has the form as expected by  $A$ . We emphasize that without the intruder's ability to apply the XOR operator and without taking into account algebraic properties of XOR, this attack could not be carried out.

### 3 The Protocol and Intruder Model

The protocol and intruder model we describe here extends standard models for the (automatic) analysis of security protocols [1,14,27,22] in two respects. First, messages can be build using the XOR operator, which is not allowed in most other protocol models. Second, in addition to the standard Dolev-Yao rewrite rules, the intruder is equipped with the above mentioned oracle rules. In what follows, we provide a formal definition of our model by defining terms, messages, protocols, the intruder, and attacks.

#### 3.1 Terms and Messages

First, recall that a finite multiset over a set  $S$  is a function  $M$  from  $S$  to  $\mathbb{N}$  with finite domain. We use the common set notation to define multisets. For

example,  $\{a, a, a, b\}$  denotes the multiset  $M$  with  $M(a) = 3$ ,  $M(b) = 1$ , and  $M(x) = 0$  for every  $x \notin \{a, b\}$ .

Terms are defined according to the following grammar:

$$term ::= \mathcal{A} \mid \mathcal{V} \mid \langle term, term \rangle \mid \{term\}_{term}^s \mid \{term\}_{\mathcal{K}}^p \mid \text{XOR}(M)$$

where  $\mathcal{A}$  is a finite set of constants (*atomic messages*), containing principal names, nonces, keys, and the constants 0 and **secret**;  $\mathcal{K}$  is a subset of  $\mathcal{A}$  denoting the set of public and private keys;  $\mathcal{V}$  is a finite set of variables; and  $M$  is a non-empty finite multiset of terms. We assume that there is a bijection  $\cdot^{-1}$  on  $\mathcal{K}$  which maps every public (private) key  $k$  to its corresponding private (public) key  $k^{-1}$ . The binary symbol  $\langle \cdot, \cdot \rangle$  is called *pairing*, the binary symbol  $\{\cdot\}^s$  is called *symmetric encryption*, the binary symbol  $\{\cdot\}^p$  is *public key encryption*. Note that a symmetric key can be any term and that for public key encryption only atomic keys (namely, public and private keys from  $\mathcal{K}$ ) can be used. A term with head XOR is called *non-standard* and otherwise it is called *standard*. Because of the algebraic properties of XOR (see below), it is convenient to define the XOR operator as done above, instead of defining it as a binary operator. We abbreviate  $\text{XOR}(\{t_1, \dots, t_n\})$  by  $\text{XOR}(t_1, \dots, t_n)$ .

Variables are denoted by  $x, y$ , terms are denoted by  $s, t, u, v$ , and finite sets of terms are written  $E, F, \dots$ , and decorations thereof, respectively. We abbreviate  $E \cup F$  by  $E, F$ , the union  $E \cup \{t\}$  by  $E, t$ , and  $E \setminus \{t\}$  by  $E \setminus t$ . The same abbreviations are used for multisets.

For a term  $t$  and a set of terms  $E$ ,  $\mathcal{V}(t)$  and  $\mathcal{V}(E)$  denote the set of variables occurring in  $t$  and  $E$ , respectively.

A *ground term* (also called *message*) is a term without variables. A (*ground*) *substitution* is a mapping from  $\mathcal{V}$  to the set of (ground) terms. The application of a substitution  $\sigma$  to a term  $t$  (a set of terms  $E$ ) is written  $t\sigma$  ( $E\sigma$ ), and is defined as usual.

Given two terms  $u, v$ , the *replacement* of  $u$  by  $v$ , denoted by  $[u \leftarrow v]$ , maps every term  $t$  to the term  $t[u \leftarrow v]$  which is obtained by replacing all occurrences of  $u$  in  $t$  by  $v$ . Note that the result of such a replacement is uniquely determined. We can compose a substitution  $\sigma$  with a replacement  $\delta$ : the substitution  $\sigma\delta$  maps every  $x \in \mathcal{V}$  to  $\sigma(x)\delta$ .

The multiset of *factors* of a term  $t$ , denoted by  $\mathcal{F}(t)$ , is recursively defined: If  $t = \text{XOR}(M)$ , then  $\mathcal{F}(t) = \uplus_{t' \in M} \mathcal{F}(t')$ , and otherwise, if  $t$  is standard,  $\mathcal{F}(t) = \{t\}$ , where  $\uplus$  is the union of multisets. Note that  $\mathcal{F}(t)$  only contains standard terms. For example, with  $a, b, c \in \mathcal{A}$ ,  $\mathcal{F}(\text{XOR}(c, \langle \text{XOR}(a, b), c \rangle, c)) = \{c, c, \langle \text{XOR}(a, b), c \rangle\}$ .

The set of *subterms* of a term  $t$ , denoted by  $\mathcal{S}(t)$ , is defined as follows:

- If  $t \in \mathcal{A}$  or  $t \in \mathcal{V}$ , then  $\mathcal{S}(t) = \{t\}$ .
- If  $t = \langle u, v \rangle$ ,  $\{u\}_v^s$ , or  $\{u\}_v^p$ , then  $\mathcal{S}(t) = \{t\} \cup \mathcal{S}(u) \cup \mathcal{S}(v)$ .
- If  $t$  is non-standard, then  $\mathcal{S}(t) = \{t\} \cup \bigcup_{u \in \mathcal{F}(t)} \mathcal{S}(u)$ .

We define  $\mathcal{S}(E) = \bigcup_{t \in E} \mathcal{S}(t)$ . Note that  $\text{XOR}(a, b) \notin \mathcal{S}(\text{XOR}(\text{XOR}(a, b), c))$ .

We define the size of a term and a set of terms basically as the size of the representation as a *Directed Acyclic Graph (DAG)*. That is, the (*DAG*) size  $|t|$  (resp.  $|E|$ ) of a term  $t$  (resp. a set of terms  $E$ ) is the cardinality of the set  $\mathcal{S}(t)$  (resp.  $\mathcal{S}(E)$ ). Note that  $|\cdot|$  applied to a set of terms will always denote the DAG size of the set rather than its cardinality.

The XOR operator is considered to be commutative, associative, nilpotent, and 0 is the unit element. According to these properties, the normal form of a term is defined as the result of the *normalization function*  $\lceil \cdot \rceil : \text{term} \rightarrow \text{term}$ . Before providing the formal definition of this function, we illustrate it by some examples:

If  $a, b, c, d \in \mathcal{A}$ , then

$$\begin{aligned} \lceil \text{XOR}(\text{XOR}(a, b, d), \text{XOR}(c, d)) \rceil &= \text{XOR}(a, b, c) \\ \lceil \text{XOR}(0, a, a, b, c), \text{XOR}(a, \text{XOR}(a, c)) \rceil &= \langle \text{XOR}(b, c), c \rangle \\ \lceil \text{XOR}(a, \langle \text{XOR}(b), a \rangle, c) \rceil &= \text{XOR}(a, \langle b, a \rangle, c). \end{aligned}$$

However,

$$\lceil \text{XOR}(\langle a, b \rangle, \langle a, c \rangle) \rceil \neq \langle 0, \text{XOR}(b, c) \rangle.$$

Formally, the normalization function is recursively defined as follows:

- $\lceil a \rceil = a$  for an atom or a variable  $a$ ,
- $\lceil \langle u, v \rangle \rceil = \langle \lceil u \rceil, \lceil v \rceil \rangle$ ,  $\lceil \{u\}_v^s \rceil = \{\lceil u \rceil\}_{\lceil v \rceil}^s$ , and  $\lceil \{u\}_v^p \rceil = \{\lceil u \rceil\}_v^p$  for terms  $u$  and  $v$ ,
- For a non-standard term  $t$ , define  $M_t$  to be the multiset of factors of  $t$  in normalized form, i.e.,

$$M_t(t') = \left( \sum_{t'', \lceil t'' \rceil = t'} \mathcal{F}(t)(t'') \right) \bmod 2$$

for every term  $t' \neq 0$ , and  $M_t(0) = 0$ . (Recall that  $\mathcal{F}(t)$  is a multiset.) Now, if  $M_t(t') = 0$  for every  $t'$ , then we set  $\lceil t \rceil = 0$ . If  $M_t(t') \neq 0$  for exactly one  $t'$ , then we define  $\lceil t \rceil = t'$ . Otherwise, we set  $\lceil t \rceil = \text{XOR}(M_t)$ .

The normalization function extends to sets, multisets of terms, and substi-

tutions in the obvious way. A term  $t$  is *normalized* if  $\overline{\overline{t}} = t$ . In the same way normalized sets, multisets of terms, and substitutions are defined. Two terms  $t$  and  $t'$  are *equivalent* (modulo XOR) if  $\overline{\overline{t}} = \overline{\overline{t'}}$ . In this case, we write  $t =_{\text{XOR}} t'$ .

One easily shows:

**Lemma 1** *For every  $n \geq 0$ , term  $t$ , and substitution  $\sigma$ :*

- (1)  $|\overline{\overline{t}}| \leq |t|$ , and
- (2)  $\overline{\overline{t\sigma}} = \overline{\overline{\overline{\overline{t}}}\sigma} = \overline{\overline{t}}\overline{\overline{\sigma}} = \overline{\overline{\overline{\overline{t}}}\overline{\overline{\sigma}}}$ .

We finally remark:

**Remark 2** *For every normalized term  $t$  with  $|t| \leq n$ , the number of arguments of XOR operators occurring in  $t$  is bounded by  $n$ . Therefore, representing  $t$  (as a DAG) needs space polynomially bounded in  $n$ .*

### 3.2 Protocols

The following definition of protocol rules, protocols, and execution orderings is explained below.

**Definition 3** *A protocol rule is of the form  $R \Rightarrow S$  where  $R$  and  $S$  are terms. A protocol  $P$  is a tuple  $(\{R_i \Rightarrow S_i, i \in \mathcal{I}\}, <_{\mathcal{I}}, E)$  where  $E$  is a finite normalized set of messages with  $0 \in E$ , the initial intruder knowledge,  $\mathcal{I}$  is a finite (index) set,  $<_{\mathcal{I}}$  is a partial ordering on  $\mathcal{I}$ , and  $R_i \Rightarrow S_i$ , for every  $i \in \mathcal{I}$ , is a protocol rule such that*

- (1) *the terms  $R_i$  and  $S_i$  are normalized;*
- (2) *for all  $x \in \mathcal{V}(S_i)$ , there exists  $j \leq_{\mathcal{I}} i$  such that  $x \in \mathcal{V}(R_j)$ ;*
- (3) *for every subterm  $\text{XOR}(t_1, \dots, t_n)$  of  $R_i$ , there exists  $k \in \{1, \dots, n\}$  such that  $\mathcal{V}(t_l) \subseteq \cup_{j <_{\mathcal{I}} i} \mathcal{V}(R_j)$  for every  $l \in \{1, \dots, n\} \setminus \{k\}$ . (Note that, since  $R_i$  is normalized,  $t_1, \dots, t_n$  are standard terms.)*

*A bijective mapping  $\pi : \mathcal{I}' \rightarrow \{1, \dots, p\}$  is called execution ordering for  $P$  if  $\mathcal{I}' \subseteq \mathcal{I}$ ,  $p$  is the cardinality of  $\mathcal{I}'$  and for all  $i, j$  we have that if  $i <_{\mathcal{I}} j$  and  $\pi(j)$  is defined, then  $\pi(i)$  is defined and  $\pi(i) < \pi(j)$ . Let  $p$  be the size of  $\pi$ .*

Given a protocol  $P$ , in the following we will assume that  $\mathcal{A}$  is the set of constants occurring in  $P$ . We define  $\mathcal{S}(P) = E \cup \cup_{i \in \mathcal{I}} (R_i \cup S_i)$  to be the set of subterms of  $P$ ,  $|P| = |\mathcal{S}(P)|$  to be the (DAG) size of  $P$ , and  $\mathcal{V} = \mathcal{V}(P)$  to be the set of variables occurring in  $P$ .

Intuitively, when executing a rule  $R_i \Rightarrow S_i$  and on receiving a (normalized) message  $m$  in a protocol run, it is first checked whether  $m$  and  $R_i$  match,

i.e., whether there exists a ground substitution  $\sigma$  such that  $m =_{\text{XOR}} R_i \sigma$ . If so,  $\lceil S_i \sigma \rceil$  is returned as output. We always assume that the messages exchanged between principals (and the intruder) are normalized — therefore,  $m$  is assumed to be normalized and the output of the above rule is not  $S_i \sigma$  but  $\lceil S_i \sigma \rceil$ . This is because principals and the intruder cannot distinguish between equivalent terms, and therefore, they may only work on normalized terms (representing the corresponding equivalence class of terms). Finally, we note that since the different protocol rules may share variables, some of the variables in  $R_i$  and  $S_i$  may be binded already by substitutions obtained from applications of previous protocol rules. We are not actually interested in a normal execution of a protocol but rather in attacks on a protocol. This is the reason why the definition of a protocol contains the initial intruder knowledge. Attacks are formally defined in Section 3.3.

Before we explain Conditions 1.–3. in Definition 3, we formalize the protocol informally described in Section 2: The set of atoms is

$$\mathcal{A} = \{na, a, I, b, ka, kb, ki, ki^{-1}, 0, \text{secret}\}$$

where in Section 2 the secret was  $N_B$ . The initial intruder knowledge is  $S_0 = \{0, I, ki, ki^{-1}, ka, kb\}$ . The protocol rules are the following, where we have only retained the rules that are used in the attack:

$$\begin{aligned} (a, 1) : & & 0 & \Rightarrow \{\langle na, a \rangle\}_{ki}^p \\ (a, 2) : & \{\langle x_{\text{secret}}, \text{XOR}(na, I) \rangle\}_{ka}^p & \Rightarrow \{x_{\text{secret}}\}_{ki}^p \\ (b, 1) : & & \{\langle x_{na}, a \rangle\}_{kb}^p & \Rightarrow \{\langle \text{secret}, \text{XOR}(x_{na}, b) \rangle\}_{ka}^p \end{aligned}$$

Obviously, Conditions 1.–3. are satisfied. We have  $\mathcal{I} = \{(a, 1), (a, 2), (b, 1)\}$  and  $\prec_{\mathcal{I}} = \{((a, 1), (a, 2))\}$ .

We now discuss the three conditions of Definition 3. Condition 1. is not a real restriction since, due to Lemma 1, the transformation performed by a protocol rule and its normalized variant coincide. Condition 2. guarantees that when with  $S_i$  an output is produced, the substitutions of all variables in  $S_i$  are determined already. Otherwise, the output of a protocol rule would be arbitrary, which is of course undesirable. For instance, if a protocol contains a protocol rule of the form  $a \Rightarrow x$  where the variable  $x$  occurs for the first time, then  $x$  can be substituted by an arbitrary message, and hence, arbitrary output can be produced by this rule. By Condition 2. such undesirable phenomena are excluded. Condition 3. guarantees that when applying a protocol rule, the substitution of the variables in this rule can uniquely and immediately be determined when given the input from the intruder for this rule. For example, if a protocol contains a rule of the form  $\text{XOR}(x, y) \Rightarrow \langle x, y \rangle$  where the variables  $x$  and  $y$  occur for the first time, then this protocol violates Condition 3: On receiving  $\text{XOR}(a, b, c)$ , for instance, infinitely many substitutions are possible,

including  $\{x \mapsto \text{XOR}(a, b), y \mapsto c\}$ ,  $\{x \mapsto \text{XOR}(b, \langle a, a \rangle), y \mapsto \text{XOR}(a, c, \langle a, a \rangle)\}$ ,  $\{x \mapsto \text{XOR}(b, \langle a, \langle a, a \rangle \rangle), y \mapsto \text{XOR}(a, c, \langle a, \langle a, a \rangle \rangle)\}$  etc. In other words, a principal can arbitrarily pick a substitution out of infinitely many possible substitutions. With Condition 3. we avoid this. The following two examples illustrate that when Condition 3. is not satisfied, then this is due to an “unreasonable” specification of the protocol.

We start with a very simple protocol which contains two rules, (A,i) and (A,j), where (A,i) precedes (A,j). Intuitively, these rules are part of a description of a principal  $A$  who would first apply (A,  $i$ ) before performing (A,  $j$ ):

$$\begin{aligned} (A, i) : \quad & \text{XOR}(x, y) \Rightarrow \dots \\ & \vdots \\ (A, j) : \quad & x \Rightarrow y \end{aligned}$$

Informally speaking, in (A,  $i$ ) principal  $A$  accepts a message which  $A$  believes to be of the form  $\text{XOR}(x, y)$ . However, since neither  $x$  nor  $y$  is known at this point,  $A$  has to accept any message. Only when performing (A,  $j$ ) one part of the message, namely  $x$ , is determined, and from this together with  $\text{XOR}(x, y)$   $A$  can compute the second part,  $y$ . The problem with the above description is that (A,  $i$ ) cannot be performed deterministically. Principal  $A$  has to guess some  $x$  and  $y$  (infinitely many substitutions are possible). Only later if (A,  $j$ ) is performed  $x$  and  $y$  can be determined. However, in an attack (A,  $j$ ) might never be applied. This phenomenon will occur in every protocol where Condition 3. is not satisfied and it is due to an ill-defined protocol specification. In the example, the way the protocol should be modeled is the following:

$$\begin{aligned} (A, i) : \quad & z \Rightarrow \dots \\ & \vdots \\ (A, j) : \quad & x \Rightarrow \text{XOR}(z, y) \end{aligned}$$

Obviously, in this formulation of the protocol Condition 3. is satisfied.

The second example presents a protocol where a message is received at one point of the protocol run and decrypted at a subsequent step. In the informal Alice and Bob notation, the protocol is stated as follows:

1.  $A \rightarrow B : \text{XOR}(\{N_a\}_K^s, \{B\}_K^s)$
2.  $B \rightarrow A : \text{XOR}(\{N_a\}_K^s, \{B\}_K^s, B)$
3.  $A \rightarrow B : K$
4.  $B \rightarrow A : N_a$

In this protocol, principal  $B$  receives  $\{N_a\}_K \oplus \{B\}_K$  from  $A$  and returns  $\{N_a\}_K^s \oplus \{B\}_K^s \oplus B$ . When  $A$  sends the key  $K$ ,  $B$  is able to retrieve  $N_a$  from the message previously received from  $A$ . The following is a naïve formalization of this protocol where the partial ordering of the rules is defined in the obvious way:

$$\begin{array}{lll}
(A, 1) : & 0 & \Rightarrow \text{XOR}(\{N_a\}_K^s, \{B\}_K^s) \\
(B, 1) : & \text{XOR}(\{x\}_y, \{B\}_y) & \Rightarrow \text{XOR}(\{x\}_y, \{B\}_y, B) \\
(A, 2) : & \text{XOR}(\{N_a\}_K^s, \{B\}_K^s, B) & \Rightarrow K \\
(B, 2) : & y & \Rightarrow x
\end{array}$$

First note that because of  $(B, 1)$ , Condition 3. is not satisfied. However, this formulation of the protocol is unrealistic because in  $(B, 1)$  principal  $B$  only accepts messages which are obtained as the xor of two encrypted messages whose key and in case of the first encrypted message whose plaintext  $B$  does not know. Hence, in a realistic implementation  $B$  is not able to check whether the received message has the required form, but rather has to accept any message. The protocol should therefore be stated as follows:

$$\begin{array}{lll}
(A, 1) : & 0 & \Rightarrow \text{XOR}(\{N_a\}_K^s, \{B\}_K^s) \\
(B, 1) : & z & \Rightarrow \text{XOR}(z, B) \\
(A, 2) : & \text{XOR}(\{N_a\}_K^s, \{B\}_K^s, B) & \Rightarrow K \\
(B, 2) : & y & \Rightarrow \{z\}_{N_b}^s \\
(B, 3) : & \text{XOR}(\{\{x\}_y^s, \{B\}_y^s\}_{N_b}^s) & \Rightarrow x
\end{array}$$

where  $N_b$  is a fresh nonce which is only known to  $B$ . In step  $(B, 2)$  and  $(B, 3)$  we describe that  $B$  receives a message  $y$ , supposedly  $K$ , which is then used to extract  $N_a$  from (the message substituted for)  $z$ . Note that this formulation of the protocol satisfies Condition 3.

We point out that in [12] no restrictions on protocol rules are put, and thus, as illustrated above, also “unreasonable” protocol specifications, where Condition 3. is not satisfied, are allowed.

### 3.3 The Intruder Model and Attacks

Our intruder model follows the Dolev-Yao intruder [13]. That is, the intruder has complete control over the network and he can derive new messages from his initial knowledge and the messages received from honest principals dur-

ing protocol runs. To derive a new message, the intruder can compose and decompose, encrypt and decrypt messages, in case he knows the key. What distinguishes the intruder we consider here from the standard Dolev-Yao intruder, is that we will equip the intruder with *guess rules*, which provide him with additional capabilities of deriving messages. In Section 3.4, we consider classes of guess rules with certain properties, so-called oracle rules. As mentioned, in Section 7 we will look at two different instances of these oracle rules, namely XOR and prefix rules.

The intruder derives new messages from a given (finite) set of message by applying intruder rules. An *intruder rule* (or *t-rule*)  $L$  is of the form  $M \rightarrow t$ , where  $M$  is a finite multiset of messages and  $t$  is a message. Given a finite set  $E$  of messages, the rule  $L$  can be applied to  $E$  if  $M$  is a subset of  $E$ , in the sense that if  $M(t') \neq 0$ , then  $t' \in E$  for every message  $t'$ . We define the *step relation*  $\rightarrow_L$  induced by  $L$  as a binary relation on (finite) sets of messages. For every finite set of messages  $E$  we have  $E \rightarrow_L E, t$  (recall that  $E, t$  stands for  $E \cup \{t\}$ ) if  $L$  is a  $t$ -rule and  $L$  can be applied to  $E$ . If  $\mathcal{L}$  denotes a (finite or infinite) set of intruder rules, then  $\rightarrow_{\mathcal{L}}$  denotes the union  $\bigcup_{L \in \mathcal{L}} \rightarrow_L$  of the step relations  $\rightarrow_L$  with  $L \in \mathcal{L}$ . With  $\rightarrow_{\mathcal{L}}^*$  we denote the reflexive and transitive closure of  $\rightarrow_{\mathcal{L}}$ .

The set of intruder rules we consider in this paper is depicted in Figure 1. In this table,  $a, b$  denote (arbitrary) messages,  $K$  is an element of  $\mathcal{K}$ , and  $E$  is a finite set of messages (considered as multiset).

We emphasize that the notion of *intruder rule* will always refer to the rules listed in Figure 1. For now, there may be any set of guess rules of the kind shown in Figure 1, later we will consider certain classes of guess rules, namely oracle rules.

We refer to certain sets of intruder rules using the notation as depicted in Figure 1. For example,  $L_{p1}(\langle a, b \rangle)$  refers to the singleton set  $\{\langle a, b \rangle \rightarrow a\}$  and  $L_c(\langle a, b \rangle)$  to the singleton set  $\{a, b \rightarrow \langle a, b \rangle\}$ . In the same way  $L_{p2}(\langle a, b \rangle)$ ,  $L_{ad}(\{a\}_K^p)$ ,  $L_{sd}(\{a\}_b^s)$ ,  $L_c(\{a\}_K^p)$ , and  $L_c(\{a\}_b^s)$  define sets consisting of one intruder rule. With  $L_{od}(a)$  and  $L_{oc}(a)$  we denote (finite or infinite) sets of guess rules. Note that, even if no guess rules are considered, i.e., for every message  $a$  the sets  $L_{od}(a)$  and  $L_{oc}(a)$  are empty, the number of decomposition and composition rules is always infinite since there are infinitely many messages  $a, b$ . The reason  $L_{p1}(\langle a, b \rangle)$ ,  $L_{p2}(\langle a, b \rangle)$ ,  $L_{ad}(\{a\}_K^p)$ ,  $L_{sd}(\{a\}_b^s)$ ,  $L_c(\langle a, b \rangle)$ ,  $L_c(\{a\}_K^p)$ , and  $L_c(\{a\}_b^s)$  denote singletons rather than intruder rules is simply notational convenience.

We further group the intruder rules as follows. In the following,  $t$  ranges over all messages.

- $L_d(t) = L_{p1}(t) \cup L_{p2}(t) \cup L_{ad}(t) \cup L_{sd}(t)$  for every message  $t$ . In case, for

instance,  $L_{p1}(t)$  is not defined, i.e., the head symbol of  $t$  is not a pair, then  $L_{p1}(t) = \emptyset$ ; analogously for the other rule sets,

- $L_d = \bigcup_t L_d(t)$ ,  $L_c = \bigcup_t L_c(t)$ ,
- $L_{od} = \bigcup_t L_{od}(t)$ ,  $L_{oc} = \bigcup_t L_{oc}(t)$ ,
- $L_o(t) = L_{oc}(t) \cup L_{od}(t)$ ,  $L_o = L_{oc} \cup L_{od}$ ,
- $\mathcal{L}_d(t)$  is the set of all decomposition  $t$ -rules in Figure 1, i.e., all  $t$ -rule in the left column of the table,
- $\mathcal{L}_d = \bigcup_t \mathcal{L}_d(t)$ ,
- $\mathcal{L}_c(t)$  is the set of all composition  $t$ -rules in Figure 1.
- $\mathcal{L}_c = \bigcup_t \mathcal{L}_c(t)$ .
- $\mathcal{L} = \mathcal{L}_d \cup \mathcal{L}_c$ .

Note that  $\mathcal{L}$  denotes the (infinite) set of all intruder rules we consider here. The set of messages the intruder can derive from a (finite) set  $E$  of messages is:

$$\text{forge}(E) = \bigcup \{E' \mid E \rightarrow_{\mathcal{L}}^* E'\}.$$

From the definition of intruder rules in Figure 1 it immediately follows:

**Lemma 4** *If  $E$  is a normalized set of messages, then  $\text{forge}(E)$  is normalized.*

The lemma says that if an intruder only sees normalized messages, then he only creates normalized messages. Intruders should be modeled in such a way that they cannot distinguish between equivalent messages since if one thinks of, for instance, the message  $\text{XOR}(a, a, b)$ , which is equivalent to  $b$ , as a bit string obtained by “XORing” the bit strings  $a$ ,  $a$ , and  $b$ , then this bit string is simply  $b$ . Therefore, in what follows we always assume that the intruder’s knowledge consists of a set of normalized messages, where every single normalized message in this set can be seen as a representative of its equivalence class.

We are now prepared to define attacks. In an attack on a protocol  $P$ , the intruder (nondeterministically) chooses some execution order for  $P$  and then tries to produce input messages for the protocol rules. These input messages are derived from the intruder’s initial knowledge and the output messages produced by executing the protocol rules. The aim of the intruder is to derive the message **secret**. If different sessions of a protocol running interleaved shall be analysed, then these sessions must be encoded into the protocol  $P$ . This is the standard approach when protocols are analysed w.r.t. a bounded number of sessions, see, for instance, [27].

**Definition 5** *Let  $P = (\{R'_j \Rightarrow S'_j \mid j \in \mathcal{I}\}, <_{\mathcal{I}}, S_0)$  be a protocol. Then an attack on  $P$  is a tuple  $(\pi, \sigma)$  where  $\pi$  is an execution ordering on  $P$  and  $\sigma$  is a normalized ground substitution of the variables occurring in  $P$  such that  $\ulcorner R_i \sigma \urcorner \in \text{forge}(\ulcorner S_0, S_1 \sigma, \dots, S_{i-1} \sigma \urcorner)$  for every  $i \in \{1, \dots, k\}$  where  $k$  is the size of  $\pi$ ,  $R_i = R'_{\pi^{-1}(i)}$ , and  $S_i = S'_{\pi^{-1}(i)}$ , and such that **secret**  $\in \text{forge}(\ulcorner S_0, S_1 \sigma, \dots, S_k \sigma \urcorner)$ .*

	Decomposition rules	Composition rules
Pair	$L_{p1}(\langle a, b \rangle): \langle a, b \rangle \rightarrow a$ $L_{p2}(\langle a, b \rangle): \langle a, b \rangle \rightarrow b$	$L_c(\langle a, b \rangle): a, b \rightarrow \langle a, b \rangle$
Asym.	$L_{ad}(\{a\}_K^p): \{a\}_K^p, K^{-1} \rightarrow a$	$L_c(\{a\}_K^p): a, K \rightarrow \{a\}_K^p$
Sym.	$L_{sd}(\{a\}_b^s): \{a\}_b^s, b \rightarrow a$	$L_c(\{a\}_b^s): a, b \rightarrow \{a\}_b^s$
Guess	$L_{od}(a): E \rightarrow a$ with $a$ subterm of $E$ and $E$ normalized.	$L_{oc}(a): E \rightarrow a$ with $E, a$ normalized and such that every proper subterm of $a$ is a subterm of $E$ .

Figure 1. Intruder Rules

Due to Lemma 1, it does not matter whether, in the above definition,  $\sigma$  is normalized or not. Also note that Lemma 4 implies:  $\lceil \text{forge}(\lceil S_0, S_1\sigma, \dots, S_{i-1}\sigma \rceil) \rceil = \text{forge}(\lceil S_0, S_1\sigma, \dots, S_{i-1}\sigma \rceil)$ .

The decision problem we are interested in is the following set of protocols:

$$\text{INSECURE} = \{P \mid \text{there exists an attack on } P\}.$$

### 3.4 Oracle Rules

Oracle rules are guess rules which satisfy certain conditions. To define these rules, we first need some new notions.

A *derivation*  $D$  of length  $n$ ,  $n \geq 0$ , is a sequence of steps of the form  $E \rightarrow_{L_1} E, t_1 \rightarrow_{L_2} \dots \rightarrow_{L_n} E, t_1, \dots, t_n$  with a finite set of messages  $E$ , messages  $t_1, \dots, t_n$ , intruder rules  $L_i \in \mathcal{L}$ , such that  $E, t_1, \dots, t_{i-1} \rightarrow_{L_i} E, t_1, \dots, t_i$  and  $t_i \notin E \cup \{t_1, \dots, t_{i-1}\}$ , for every  $i \in \{1, \dots, n\}$ . The rule  $L_i$  is called the  *$i$ th rule* in  $D$  and the step  $E, t_1, \dots, t_{i-1} \rightarrow_{L_i} E, t_1, \dots, t_i$  is called the  *$i$ th step* in  $D$ . We write  $L \in D$  to say that  $L \in \{L_1, \dots, L_n\}$ . If  $S$  is a set of intruder rules, then we write  $S \notin D$  to say  $S \cap \{L_1, \dots, L_n\} = \emptyset$ . The message  $t_n$  is called the *goal* of  $D$ .

We also need *well-formed* derivations which are derivations where every message generated by an intermediate step either occurs in the goal or in the initial set of messages.

**Definition 6** *Let  $D = E \rightarrow_{L_1} \dots \rightarrow_{L_n} E'$  be a derivation with goal  $t$ . Then,  $D$  is well-formed if for every  $L \in D$  and  $t'$  we have that  $L \in \mathcal{L}_c(t')$  implies*

$t' \in \mathcal{S}(E, t)$ , and  $L \in \mathcal{L}_d(t')$  implies  $t' \in \mathcal{S}(E)$ .

We can now define oracle rules. Condition 1. in the following definition will allow us to bound the length of derivations. Condition 2. says that to derive  $a$  from  $E$  it is not necessary to first compose a message  $t$  from  $E$  using an oracle rule and then decompose  $t$  to obtain  $a$ . Condition 3. allows us to replace a message  $u$  which can be composed from  $F \setminus u$  by a smaller message  $\epsilon(u)$  in an application of an oracle rule. Conditions 2. and 3. together are later used (see Section 6) to bound the size of the substitution  $\sigma$  of an attack. They allow us to replace a subterm  $u$  in  $\sigma$ , composed by the intruder, by a smaller message.

**Definition 7** Let  $L_o = L_{oc} \cup L_{od}$  be a (finite or infinite) set of guess rules, where  $L_{oc}$  and  $L_{od}$  denote disjoint sets of composition and decomposition guess rules, respectively. Then,  $L_o$  is a set of oracle rules (w.r.t.  $L_c \cup L_d$  as defined above) iff:

- (1) For every message  $t$ , if  $t \in \text{forge}(E)$ , then there exists a well-formed derivation from  $E$  with goal  $t$ .
- (2) If  $F \rightarrow_{L_{oc}(t)} F, t$  and  $F, t \rightarrow_{L_d(t)} F, t, a$ , then there exists a derivation  $D$  from  $F$  with goal  $a$  such that  $L_d(t) \notin D$ .
- (3) For every non atomic message  $u$ , there exists a normalized message  $\epsilon(u)$  with  $|\epsilon(u)| < |u|$  such that: For every finite set  $F$  of messages with  $0 \in F$ , if  $F \setminus u \rightarrow_{L_c(u)} F$ , i.e.,  $u$  can be composed from  $F \setminus u$  in one step, then  $F \rightarrow_{L_o(t)} F, t$  implies  $\lceil t[u \leftarrow \epsilon(u)] \rceil \in \text{forge}(\lceil F[u \leftarrow \epsilon(u)] \rceil)$  and  $\epsilon(u) \in \text{forge}(F)$  for every message  $t$ .

In Section 7 we will present sets of oracle rules. An example for guess rules which do not form a set of oracle rules is the following:  $L_{od}(t) = \{\{t\}_c^s \rightarrow t\}$  and  $L_{oc}(\{t\}_{\langle a, b \rangle}^s) = \{\{t\}_{\langle a, b \rangle}^s \rightarrow \{t\}_c^s\}$  where  $t$  is an arbitrary message and  $a, b, c$  are fixed atomic messages. That is,  $L_{oc}(\{t\}_{\langle a, b \rangle}^s)$  allows to turn an encryption with key  $\langle a, b \rangle$  into an encryption with key  $c$ . Using  $L_{od}(t)$  such a message can then be decrypted. It is easy to check that neither of the three conditions in Definition 7 is satisfied.

## 4 Main Theorem and the NP Decision Algorithm

We now state the main theorem of this paper. In Section 7, this theorem will allow us to show that INSECURE is in NP in presence of an intruder that uses XOR rules and prefix rules, respectively.

**Theorem 8** Let  $L_o$  be a set of oracle rules. If  $E \rightarrow t \in^? L_o$  can be checked in polynomial time in  $|E, t|$  for every finite set  $E$  of messages and message  $t$ , then INSECURE is in NP.

Input: protocol  $P = (\{R_\iota \Rightarrow S_\iota, \iota \in \mathcal{I}\}, \prec_{\mathcal{I}}, S_0)$  with  $n = |P|$ ,  $V = \text{Var}(P)$ .

- (1) Guess an execution order  $\pi$  for  $P$ . Let  $k$  be the size of  $\pi$ .  
Let  $R_i = R'_{\pi^{-1}(i)}$  and  $S_i = S'_{\pi^{-1}(i)}$  for  $i \in \{1, \dots, k\}$
- (2) Guess a normalized ground substitution  $\sigma$  such that  $|\{\sigma(x) \mid x \in V\}| \leq 3n$ .
- (3) Check that  $\lceil R_i \sigma \rceil \in \text{forge}(\lceil \{S_j \sigma \mid j < i\} \cup \{S_0\} \rceil)$  for every  $i \in \{1, \dots, k\}$ .
- (4) Check  $\text{secret} \in \text{forge}(\lceil \{S_j \sigma \mid j < k + 1\} \cup \{S_0\} \rceil)$ .
- (5) If each check is successful, then answer “yes”, and otherwise, “no”.

Figure 2. NP Decision Procedure for Insecurity

The NP decision procedure is given in Figure 2. In the following two sections, we show that this procedure is sound and complete, and that it runs in non-deterministic polynomial time. From this, Theorem 8 follows immediately.

Clearly, the procedure is sound. In Section 5 we show that the procedure runs in non-deterministic polynomial time. To this end, we prove that the derivation problem (called ground reachability problem in [12]) can be decided in polynomial time (Theorem 9). This result is of independent interest. As a corollary, we obtain the desired complexity bound (Corollary 10). Completeness of our procedure is then established in Section 6 where we show that if there exists an attack  $(\pi, \sigma)$  on  $P$ , then there is one with the size of  $\sigma$  bounded as in step 2. of the procedure (Theorem 20).

## 5 Deciding the Derivation Problem

The derivation problem is defined as follows:

$$\text{DERIVE} = \{(E, t) \mid t \in \text{forge}(E)\}$$

where  $E$  is a finite set of messages and  $t$  is a message, both given as DAGs.

We show:

**Theorem 9**  $\text{DERIVE} \in \text{PTIME}$  given that  $E \rightarrow t \in^? L_o$  can be checked in polynomial time in  $|E, t|$  for every finite set  $E$  of messages and message  $t$ .

*Proof.* Let  $d_t(E)$  be the set consisting of the messages in  $E$  and the messages  $t' \in \mathcal{S}(E, t)$  that can be derived from  $E$  in one step. Using that the number of terms  $t' \in \mathcal{S}(E, t)$  is linear in  $|E, t|$  and that  $E \rightarrow t \in^? L_o$  can be checked in polynomial time it is easy to see that  $d_t(E)$  can be computed in polynomial time in  $|E, t|$ . Now, if  $t \in \text{forge}(E)$ , then Definition 7 guarantees that there

exists a well-formed derivation  $D = E \rightarrow_{L_1} E, t_1 \rightarrow \dots \rightarrow_{L_r} E, t_1, \dots, t_r$ , with  $t_r = t$ . In particular,  $t_i \in \text{Sub}(E, t)$  for every  $i \in \{1, \dots, k\}$ . By definition of derivations, all  $t_i$  are different. It follows  $r \leq |t, E|$ . Moreover, with  $d_t^0(E) = E$  and  $d_t^{l+1}(E) = d_t(d_t^l(E))$  we have that  $t \in d_t^{|E, t|}(E)$  iff  $t \in \text{forge}(E)$ . Since  $d_t^{|E, t|}(E)$  can be computed in polynomial time, Theorem 9 follows.  $\square$

As an immediate consequence we obtain:

**Corollary 10** *The procedure depicted in Figure 2 runs in polynomial time in  $|P|$  given that  $E \rightarrow t \in^? L_o$  can be checked in polynomial time in  $|E, t|$  for every finite set  $E$  of messages and message  $t$ .*

*Proof.* It suffices to observe that the steps 3. and 4. of our procedure can be performed in deterministic polynomial time in  $|P|$ . Given  $\sigma$  with  $|\{\sigma(x) \mid x \in V\}| \leq 3n$  and using Lemma 1 we have that  $|\lceil R_i \sigma, S_0 \sigma, \dots, S_{i-1} \sigma \rceil| \leq |R_i \sigma, S_0 \sigma, \dots, S_{i-1} \sigma| \leq |R_i, S_0, \dots, S_{i-1}, \sigma(\mathcal{V})| \leq |P| + 3 \cdot |P| \leq 4 \cdot |P|$ . Similarly for  $|\lceil \text{secret}, S_0 \sigma, \dots, S_k \sigma \rceil|$ . Hence, using Theorem 9 it follows that step 3.-4. of our procedure can be performed in deterministic polynomial time in  $|P|$ .  $\square$

## 6 Linear Bounds on Attacks

We now show that the size of an attack can be bounded as required in step 2. of the algorithm depicted in Figure 2.

In what follows, we assume that  $L_o$  is a set of oracle rules. If  $t \in \text{forge}(E)$ , we denote by  $D_t(E)$  a well-formed derivation from  $E$  with goal  $t$  (chosen arbitrarily among the possible ones). Note that there always exists such a derivation since the definition of oracle rules ensures that a well-formed derivation exists iff a derivation exists.

**Definition 11** *Let  $P = (\{R_i \Rightarrow S_i, i \in \mathcal{I}\}, \langle_{\mathcal{I}}, S_0)$  be a protocol. An attack  $(\pi, \sigma)$  is normal if  $|\sigma| = \sum_{x \in \mathcal{V}(P)} |\sigma(x)|$  is minimal.*

Clearly, if there is an attack, there is a normal attack. Note, however, that normal attacks are not necessarily uniquely determined.

In Lemma 19 we prove, using Lemma 13 to 18, that normal attacks can always be constructed by linking subterms that are initially occurring in the problem specification. This will allow us to bound the size of attacks as desired (Theorem 20 and Corollary 10).

Let  $P = (\{R_j \Rightarrow S_j, j \in \mathcal{I}\}, \langle_{\mathcal{I}}, S_0)$  be a protocol such that  $(\pi, \sigma)$  is an attack on  $P$ . Let  $k$  be the size of  $\pi$ . We define  $R_i = R'_{\pi^{-1}(i)}$  and  $S_i = S'_{\pi^{-1}(i)}$

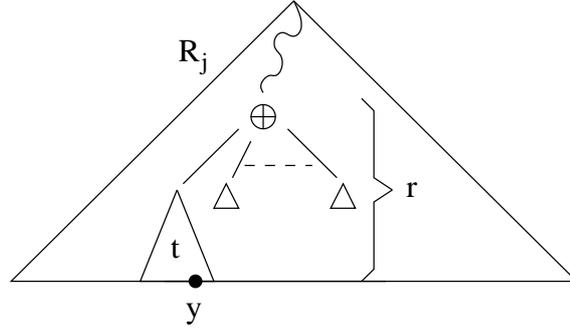


Figure 3. Structure of  $R_j$

for  $i \in \{1, \dots, k\}$ . Recall that  $\mathcal{S}(P)$  is the set of subterms of  $P$ ,  $\mathcal{A} \subseteq \mathcal{S}(P)$ , and  $\mathcal{V} = \mathcal{V}(\mathcal{S}(P))$  is the set of variables occurring in the protocol.

**Definition 12** *Let  $t$  and  $t'$  be two terms and  $\theta$  a ground substitution. Then,  $t$  is a  $\theta$ -match of  $t'$ , denoted  $t \sqsubseteq_{\theta} t'$ , if  $t$  and  $t'$  are standard,  $t$  is not a variable, and  $\lceil t\theta \rceil = t'$ .*

The following lemma says that standard subterms  $s$  occurring in the substitution  $\sigma$  of a normal attack start with a subterm of the protocol under consideration or (at least) occur on the left hand-side of a protocol rule when the substitution is applied. Note that even if  $s$  occurs in  $\sigma(x)$  and  $x$  occurs in  $R_i$ ,  $s$  does not need to occur in  $\lceil R_i\sigma \rceil$  because of the normalization.

**Lemma 13** *If  $(\pi, \sigma)$  is a normal attack, then for all  $i \in \{1, \dots, k\}$ ,  $x \in \mathcal{V}(R_i)$ , and standard subterms  $s$  of  $\sigma(x)$ , there exists  $j \leq i$  such that  $s \in \mathcal{S}(\lceil R_j\sigma \rceil)$  or there exists  $t \in \mathcal{S}(P)$  with  $t \sqsubseteq_{\sigma} s$ .*

*Proof.* Assume that there exists  $i \in \{1, \dots, k\}$ ,  $x \in \mathcal{V}(R_i)$ , and  $s$  standard subterm of  $\sigma(x)$  such that for all  $j \leq i$ :  $s \notin \mathcal{S}(\lceil R_j\sigma \rceil)$ . Define

$$j = \min\{i' \mid y \in \mathcal{V}(R_{i'}) \text{ and } s \text{ subterm of } \sigma(y)\}$$

and let  $y$  be a variable of  $R_j$  such that  $s$  is a subterm of  $\sigma(y)$ . Note that  $j \leq i$ , and thus,  $s \notin \mathcal{S}(\lceil R_j\sigma \rceil)$ . Let  $S_{y,s}$  be the set of subterms  $t$  of  $R_j$  such that  $y \in \mathcal{V}(t)$  and  $s$  is a subterm of  $\lceil t\sigma \rceil$ . This subset contains  $y$ , and thus, is not empty. Let  $t \in S_{y,s}$  be maximal in  $S_{y,s}$  w.r.t. the subterm ordering. We know that  $t \neq R_j$ . Let  $r \in \mathcal{S}(R_j)$  with  $t \in \mathcal{S}(r)$  and there exists no  $r' \in \mathcal{S}(r)$  with  $t \in \mathcal{S}(r')$ . Then, since  $s \notin \mathcal{S}(\lceil r\sigma \rceil)$ ,  $r$  must be of the form  $\text{XOR}(t, t_1, \dots, t_n)$  with  $t, t_1, \dots, t_n$  standard (since  $R_j$  is normalized) and  $n \geq 1$  such that there exists  $i \in \{1, \dots, n\}$ , say  $i = 1$ , with  $\lceil t\sigma \rceil = \lceil t_1\sigma \rceil$  ( $s$  has been eliminated by normalization). In particular,  $s \in \mathcal{S}(\lceil t_1\sigma \rceil)$ . (This situation is depicted in Figure 3.)

Let  $M_{s,t_1}$  be the set of subterms  $t'$  of  $t_1$  such that  $s \in \mathcal{S}(\lceil t'\sigma \rceil)$ , and let  $t_s$  be minimal in  $M_{s,t_1}$  w.r.t. the subterm ordering. By Definition 3, (3), and since

$y$  first appears in  $R_j$ , we have that for all  $z \in \mathcal{V}(t_1)$ , there exists  $j_z < j$  with  $z \in \mathcal{V}(R_{j_z})$ . Hence, by minimality of  $j$ ,  $s$  is not a subterm of  $\{\sigma(z) \mid z \in \mathcal{V}(t_1)\}$ , and thus,  $t_s \notin \mathcal{V}$ . Moreover,  $t_s$  is standard by minimality (otherwise, since  $s$  is standard, there would be a factor  $t'_s$  of  $t_s$  such that  $s \in \mathcal{S}(\ulcorner t'_s \sigma \urcorner)$ ). Together, this implies  $\ulcorner t_s \sigma \urcorner = s$  and  $t_s \sqsubseteq_\sigma s$  (recall that  $t_s$  is minimal).  $\square$

Roughly speaking, the following Lemma 14 states that if a term  $\gamma$  can be forged from a set of messages  $E$  by composing with  $L_c$ , say composing two messages  $\gamma_1$  and  $\gamma_2$  both derived from  $E$ , then it is always possible to avoid decomposing  $\gamma$  with  $L_d$  in a derivation from  $E$  with goal  $t$  for some  $t$  since such a decomposition would generate a message  $\gamma_1$  or  $\gamma_2$  that can be derived from  $E$  in another way.

First, we need some notation: If  $D_1 = E_1 \rightarrow \dots \rightarrow F_1$  and  $D_2 = E_2 \rightarrow \dots \rightarrow F_2$  are two derivations such that  $E_2 \subseteq F_1$ , then  $D = D_1.D_2$  is defined as the *concatenation* of the steps of  $D_1$  and the ones in  $D_2$ . In addition, to obtain a derivation, we delete in  $D$  the steps from  $D_2$  that generate terms already present in  $F_1$ .

**Lemma 14** *Let  $t \in \text{forge}(E)$  and  $\gamma \in \text{forge}(E)$  be given with a derivation  $D_\gamma$  from  $E$  ending with an application of a rule in  $\mathcal{L}_c$ . Then, there is a derivation  $D'$  from  $E$  with goal  $t$  satisfying  $L_d(\gamma) \notin D'$ .*

*Proof.* By definition of a derivation,  $L_d(\gamma) \notin D_\gamma$ . Let  $D$  be  $D_\gamma$  without its last rule, i.e.,  $D_\gamma$  is  $D$  followed by some  $L \in \mathcal{L}_c$ . Define  $D'' = D.D_t(E) = D.D'''$  —  $D'''$  is obtained from  $D_t(E)$  by removing redundant steps. Note that  $D''$  is a derivation with goal  $t$ . We distinguish two cases:

Assume  $L = L_c(\gamma)$ . Then  $L_d(\gamma) \notin D''$  since the (two) direct subterms of  $\gamma$  are created in  $D$ , and thus,  $L_d(\gamma) \notin D''$ . In other words,  $D' = D''$  is the derivation we are looking for.

Assume  $L = L_{oc}(\gamma)$ . Then, if  $L_d(\gamma) \notin D''$  setting  $D' = D''$  we are done. Otherwise, let  $F_1$  be the final set of messages of  $D$ . Now, Definition 7, (2) implies that every step in  $D'''$  of the form  $F_1, F_2, \gamma \rightarrow_{L_d(\gamma)} F_1, F_2, \gamma, \beta$  can be replaced by a derivation from  $F_1, F_2$  with goal  $\beta$  that does not contain rules from  $L_d(\gamma)$ . Replacing steps in this way and then removing redundant steps yields the derivation  $D'$  we are looking for.  $\square$

The proof of the following lemma is trivial.

**Lemma 15** *For every normalized finite set  $E$  of messages, message  $t$ , and  $t$ -rule  $L$ , if  $E \rightarrow_L E, t$  then all proper subterms of  $t$  are subterms of  $E$ .*

*Proof.* For  $L \in L_{od} \cup L_{oc}$  use the definition of decomposition and composition guess rules. For  $L \in L_d \cup L_c$  the statement is obvious.  $\square$

The next lemma states that if a term  $t'$  is a subterm of a term  $t$  and this term is derived from a set  $E$  but  $t'$  is not a subterm of  $E$ , then  $t'$  can be derived from  $E$  and the last step of the derivation is a composition rule.

**Lemma 16** *Assume that  $t' \in \mathcal{S}(t) \setminus \mathcal{S}(E)$  and  $t \in \text{forge}(E)$ , then  $t' \in \text{forge}(E)$  and there exists a (well-formed) derivation from  $E$  with goal  $t'$  ending with a composition rule.*

*Proof.* Let  $D = E_0 \rightarrow_{L_1} E_1 \cdots \rightarrow_{L_n} E_n$  be a derivation of  $t$  from  $E_0 = E$ . Then, there exists a least  $i \neq 0$  such that  $t' \in \mathcal{S}(E_i)$  since  $t'$  is a subterm of  $E_n$ . Assume that  $L_i$  is an  $s$ -rule for some  $s$ . Then,  $t'$  is a subterm of  $s$ . If  $t'$  is a proper subterm of  $s$ , Lemma 15 implies that  $t'$  is a subterm of  $E_{i-1}$  in contradiction to the minimality of  $i$ . Thus,  $t' = s$  and therefore,  $t' \in \text{forge}(E)$ . By the definition of oracle rules, there exists a well-formed derivation  $D'$  of  $t'$ . If the last step in this derivation is a decomposition rule, then this implies  $t' \in \mathcal{S}(E)$  in contradiction to the assumption. Thus, the last step of  $D'$  is a composition rule.  $\square$

The subsequent lemma will allow us to replace certain subterms occurring in a substitution of an attack by smaller terms. Note that from the assumption made in this lemma it follows that  $s$  can be derived from  $E$  such that the last rule is a composition rule. This allows to replace  $s$  by a smaller term since when deriving  $t$ , decomposing  $s$  will not be necessary.

**Lemma 17** *Let  $E$  and  $F$  be two sets of normalized messages such that  $0 \in E \cup F$ . Let  $t \in \text{forge}(E, F)$  and  $s \in \text{forge}(E)$  non atomic such that  $s \notin \mathcal{S}(E)$ . Finally, let  $\delta$  be the replacement  $[s \leftarrow \epsilon(s)]$ , where  $\epsilon(s)$  is defined as in Definition 7. Then,  $\lceil t\delta \rceil \in \text{forge}(\lceil E\delta, F\delta \rceil)$ .*

*Proof.* Let  $D_s = E \rightarrow_{L_1} E, t_1 \rightarrow_{L_2} \dots \rightarrow_{L_p} E, t_1, \dots, t_p \rightarrow_{L_c(s)} E, t_1, \dots, t_p, s$ . By induction on  $i$  and using Lemma 15, it follows that all proper subterms of  $t_i$  are subterms of  $E, t_1, \dots, t_{i-1}$ . Using  $s \notin \mathcal{S}(E)$  and  $s \neq t_i$ , this implies  $s \notin \mathcal{S}(t_i)$ , and thus,  $\lceil t_i\delta \rceil = t_i$  (note that  $t_i$  is normalized) and  $\lceil t_i\delta \rceil \in \text{forge}(\lceil E\delta \rceil)$ , for every  $i \in \{1, \dots, p\}$ . Thanks to Lemma 14, there exists a derivation  $D = E, F, t_1, \dots, t_p \rightarrow_{L_{p+1}} E, F, t_1, \dots, t_{p+1} \rightarrow \dots \rightarrow_{L_n} E, F, t_1, \dots, t_n$  with  $t_n = t$  and  $L_i \notin L_d(s)$ , for every  $i \in \{p+1, \dots, n\}$ . We know  $\lceil t_i\delta \rceil \in \text{forge}(\lceil E\delta \rceil)$ , for every  $i \in \{1, \dots, p\}$ . We show by induction on  $i$ ,  $p \leq i \leq n$ , that  $\lceil t_i\delta \rceil \in \text{forge}(\lceil E\delta, F\delta \rceil)$ . For  $i = p$  this is by Definition 7, (4). Assume that  $i > p$  and the property is true for all  $j < i$ . Then we have three cases:

- If  $L_i = L_c(\langle a, b \rangle)$ , then either  $t_i = s$ , and thus, using the definition of  $\epsilon(s)$ ,  $\lceil t_i\delta \rceil = \epsilon(s) \in \text{forge}(\lceil E\delta, F\delta \rceil)$ , or  $\lceil t_i\delta \rceil = \langle \lceil a\delta \rceil, \lceil b\delta \rceil \rangle \in \text{forge}(\lceil E\delta, F\delta \rceil)$  since  $\{a, b\} \subseteq E \cup F \cup \{t_1, \dots, t_{i-1}\}$ . Analogously for  $\{a\}_b^s$  and  $\{a\}_K^p$ .
- If  $L_i = L_{p1}(\langle t_i, a \rangle)$ , then  $s \neq \langle t_i, a \rangle$  since  $L_i \notin L_d(s)$ , and therefore,

$\ulcorner t_i \delta^\flat \in \text{forge}(\ulcorner \langle t_i, a \rangle \delta^\flat) \subseteq \text{forge}(\ulcorner E \delta, F \delta^\flat)$  since  $\langle t_i, a \rangle \in E \cup F \cup \{t_1, \dots, t_{i-1}\}$ . Analogously for  $L_{p2}$ ,  $L_{sd}$ , and  $L_{ad}$ .

- If  $L_i \in L_{oc} \cup L_{od}$ , then thanks to Definition 7, (4), we have:  $\ulcorner t_i \delta^\flat \in \text{forge}(\ulcorner E \delta, F \delta, t_1 \delta, \dots, t_{i-1} \delta^\flat)$  and thus:  $\ulcorner t_i \delta^\flat \in \text{forge}(\ulcorner E \delta, F \delta^\flat)$ .

For  $i = n$ , this gives us  $\ulcorner t \delta^\flat \in \text{forge}(\ulcorner E \delta, F \delta^\flat)$ .  $\square$

The next lemma will be used to remove one application of the normalization function.

**Lemma 18** *Let  $\sigma$  be a normalized ground substitution,  $E$  a set of normalized terms,  $s$  a normalized standard non atomic term, and  $\delta$  the replacement  $[s \leftarrow \epsilon(s)]$ . Let  $\sigma' = \sigma \delta$ . If there is no standard subterm  $t$  of  $E$  such that  $t \sqsubseteq_\sigma s$ , then  $\ulcorner E \sigma^\flat = \ulcorner E \sigma^\flat \delta^\flat$ .*

*Proof.* Since there is no standard subterm  $t'$  of  $E$  such that  $t' \sqsubseteq_\sigma s$ , we have  $(E\sigma)\delta = E(\sigma\delta)$  and therefore  $\ulcorner E \sigma^\flat = \ulcorner (E\sigma) \delta^\flat$ . Let us prove, by induction on the structure of terms, that for all  $t \in \mathcal{S}(E)$ , we have  $\ulcorner t \sigma^\flat = \ulcorner t \sigma^\flat \delta^\flat$ . This will conclude the proof of the lemma.

- If  $t \in \mathcal{A}$ , then  $t \neq s$  by assumption. Thus,  $\ulcorner t \sigma^\flat \delta^\flat = t = \ulcorner t \sigma^\flat$ .
- If  $t \in \mathcal{V}$ , then  $\ulcorner t \sigma^\flat = t\sigma$ , and therefore,  $\ulcorner t \sigma^\flat = \ulcorner (t\sigma) \delta^\flat = \ulcorner t \sigma^\flat \delta^\flat$ .
- If  $t = \langle v, w \rangle$ , we have  $s \neq \langle \ulcorner v \sigma^\flat, \ulcorner w \sigma^\flat \rangle$  since otherwise  $t \sqsubseteq_\sigma s$ , and  $\ulcorner t \sigma^\flat = \langle \ulcorner v \sigma^\flat, \ulcorner w \sigma^\flat \rangle$ . By induction, this gives  $\ulcorner t \sigma^\flat = \langle \ulcorner v \sigma^\flat \delta^\flat, \ulcorner w \sigma^\flat \delta^\flat \rangle$ , and therefore,  $\ulcorner t \sigma^\flat = \ulcorner \langle v \sigma, w \sigma \rangle^\flat \delta^\flat = \ulcorner t \sigma^\flat \delta^\flat$  since  $s \neq \ulcorner t \sigma^\flat$ . The cases  $t = \{u\}_v^s$  and  $t = \{u\}_K^p$  are similar.
- If  $t = \text{XOR}(\mathcal{T})$ , where  $\mathcal{T}$  is a multiset of standard terms, we have:

$$\begin{aligned}
\ulcorner t \sigma^\flat &= \ulcorner \text{XOR}(\{t' \sigma' \mid t' \in \mathcal{T}\})^\flat \\
&= \ulcorner \text{XOR}(\{\ulcorner t' \sigma^\flat \mid t' \in \mathcal{T}\})^\flat && \text{(Definition of } \ulcorner \cdot \text{)} \\
&= \ulcorner \text{XOR}(\{\ulcorner t' \sigma^\flat \delta^\flat \mid t' \in \mathcal{T}\})^\flat && \text{(By induction on every } t' \in \mathcal{T}\text{)} \\
&= \ulcorner \text{XOR}(\{\ulcorner t' \sigma^\flat \delta \mid t' \in \mathcal{T}\})^\flat && \text{(Definition of } \ulcorner \cdot \text{)} \\
&= \ulcorner \text{XOR}(\{\ulcorner t' \sigma^\flat \mid t' \in \mathcal{T}\})^\flat \delta^\flat && \text{(Definition of } \delta \text{ and } s \text{ standard)} \\
&= \ulcorner t \sigma^\flat \delta^\flat && \text{(Definition of } \ulcorner \cdot \text{)}
\end{aligned}$$

$\square$

The main lemma, which shows that a substitution of a normal attack can be build up from subterms of terms occurring in  $P$ , is proved next.

**Lemma 19** *Given a normal attack  $(\pi, \sigma)$ , for all variables  $x$  and for all factors  $v_x$  of  $\sigma(x)$ , there exists  $t \in \mathcal{S}(P)$  such that  $t \sqsubseteq_\sigma v_x$ .*

*Proof.* Assume that (\*): For every  $t$ ,  $t \sqsubseteq_{\sigma} v_x$  implies  $t \notin \mathcal{S}(P)$ . We will lead this to a contradiction. Since  $\mathcal{A} \subseteq \mathcal{S}(P)$ , we have  $v_x \notin \mathcal{A}$ , and since  $v_x$  is a factor of  $\sigma(x)$ ,  $v_x$  is standard. By Lemma 13 and (\*), there exists  $j$  such that  $v_x \in \mathcal{S}(\lceil R_j \sigma \rceil)$ . Let  $N_x$  be minimal among the possible  $j$ . If  $v_x \in \mathcal{S}(\lceil S_i \sigma \rceil)$  for some  $i$ , (\*) implies that there exists  $y \in \mathcal{V}(S_i)$  with  $v_x \in \mathcal{S}(\sigma(y))$ . Then, by Definition 3, (2) there exists  $R_{i'}, i' \leq i$  such that  $y \in \mathcal{V}(R_{i'})$ . Thus, Lemma 13 and (\*) imply that there exists  $j \leq i$  with  $v_x \in \mathcal{S}(\lceil R_j \sigma \rceil)$ . Note also that  $v_x \notin \mathcal{S}(S_0)$  since otherwise  $v_x \in \mathcal{S}(P)$ . Now, the minimality of  $N_x$  yields  $i \geq N_x$ . Summarizing, we have:  $v_x$  is not a subterm of  $E_0 = \lceil S_0 \sigma, \dots, S_{N_x-1} \sigma \rceil$ , and  $v_x$  is a subterm of  $\lceil R_{N_x} \sigma \rceil$ . Thus, by Lemma 16,  $v_x \in \text{forge}(E_0)$ .

Let us define the replacement  $\delta = [v_x \leftarrow \epsilon(v_x)]$  where  $\epsilon(v_x)$  is defined as in Definition 7. Since  $(\pi, \sigma)$  is an attack, for all  $j$ , we have:

$$\lceil R_j \sigma \rceil \in \text{forge}(\lceil S_0 \sigma, \dots, S_{j-1} \sigma \rceil)$$

We distinguish two cases: Assume first  $j < N_x$ . Then, by minimality of  $N_x$ ,  $v_x$  is neither a subterm of  $\lceil R_j \sigma \rceil$  nor a subterm of  $\lceil S_0 \sigma, \dots, S_{j-1} \sigma \rceil$ . With  $\lceil R_j \sigma \rceil \in \text{forge}(\lceil S_0 \sigma, \dots, S_{j-1} \sigma \rceil)$  it follows  $\lceil R_j \sigma \rceil \delta \in \text{forge}(\lceil S_0 \sigma \delta, \dots, S_{j-1} \sigma \delta \rceil)$ . Assume now that  $j \geq N_x$ . With  $t = \lceil R_j \sigma \rceil$ ,  $s = v_x$ ,  $E = E_0$ , and  $F = \lceil S_{N_x} \sigma, \dots, S_{j-1} \sigma \rceil$ , Lemma 17 implies  $\lceil R_j \sigma \rceil \delta \in \text{forge}(\lceil S_0 \sigma \delta, \dots, S_{j-1} \sigma \delta \rceil)$ . Thus, in both cases

$$\lceil R_j \sigma \rceil \delta \in \text{forge}(\lceil S_0 \sigma \delta, \dots, S_{j-1} \sigma \delta \rceil).$$

Now, with  $E = \{S_0, \dots, S_{j-1}\}$  and  $E = \{R_j\}$ , respectively, (\*) and Lemma 18 imply for all  $j$ :  $\lceil R_j \sigma \rceil \in \text{forge}(\lceil S_0 \sigma', \dots, S_{j-1} \sigma' \rceil)$ , where  $\sigma' = \sigma \delta$ . Hence,  $(\pi, \sigma')$  is an attack. But since  $\sigma'$  is obtained from  $\sigma$  by replacing  $v_x$  by a strictly smaller message, namely  $\epsilon(v_x)$ , we obtain  $|\sigma'| < |\sigma|$ , a contradiction to the assumption that  $(\pi, \sigma)$  is a normal attack.  $\square$

We can now use this lemma to bound the size of every  $\sigma(x)$ :

**Theorem 20** *For every protocol  $P$ , if  $(\pi, \sigma)$  is a normal attack on  $P$ , then  $|\{\sigma(x) \mid x \in \mathcal{V}\}| \leq 3 \cdot |P|$ , where  $|P|$  is the size of  $P$  as defined in Section 3.2.*

*Proof.* Let  $F = \{s \mid \exists x \in \mathcal{V}, s \in \mathcal{F}(\sigma(x))\}$  For every  $s$  in the set  $F$  we introduce a new variable  $x_s$  and we define a substitution  $\sigma'$  such that  $\sigma'(x_s) = s$  (and other variables are mapped to themselves). Let  $\mathcal{V}' = \{x_s\}_{s \in F}$ . The cardinality  $\text{Card}(\mathcal{V}')$  of  $\mathcal{V}'$  can be bounded as follows:

*Claim.*  $\text{Card}(\mathcal{V}') \leq |P|$

*Proof of the claim.* We define a function  $f : \mathcal{V}' \rightarrow \mathcal{S}(P)$  as follows. Due to Lemma 19, for every  $y \in \mathcal{V}'$ , there exists  $t_y \in \mathcal{S}(P)$  such that  $t_y \notin \mathcal{V}$  and  $\lceil t_y \sigma \rceil = \sigma'(y)$ . We define  $f(y) = t_y$ . The function  $f$  is injective since  $t_s = t_{s'}$

implies  $\lceil t_s \sigma \rceil = \lceil t_{s'} \sigma \rceil$ . Thus,  $\text{Card}(\mathcal{V}') \leq |\mathcal{S}(P)| = |P|$ , which concludes the proof of the claim.

Let  $S = F \cup \{\sigma(x) \mid x \in \mathcal{V}\}$ . For all  $x \in \mathcal{V}$  let  $\sigma''(x) = \lceil \text{XOR}(x_{s_1}, \dots, x_{s_m}) \rceil$  with  $\{s_1, \dots, s_m\} = \mathcal{F}(\sigma(x))$ . Note that, since the  $s$ 's are normalized standard messages,  $\sigma(x) = \sigma'(\sigma''(x))$ .

$$\lceil \lceil t \sigma'' \rceil \sigma' \rceil = \lceil t \sigma \rceil \quad \text{for all } t$$

Hence  $\lceil t \sigma'' \rceil$  is a  $\sigma'$ -match of  $\lceil t \sigma \rceil$ . However  $\lceil t \sigma'' \rceil \sigma'$  is not necessarily normalized which might be problematic for the sequel. Hence let us build another term  $\lceil t \sigma'' \rceil \sigma'$  from  $\lceil t \sigma'' \rceil$ , such that  $\lceil t \sigma'' \rceil \sigma' = \lceil t \sigma \rceil$ . Intuitively, it amounts to eliminate all subterms that “would get deleted” in  $\lceil t \sigma'' \rceil$  by normalizing  $\lceil t \sigma'' \rceil \sigma'$ . Let us define :

$$\text{XOR}(v_1, \dots, v_n)^{\sigma'} = \begin{cases} \text{XOR}(v_1, \dots, v_n \setminus v_i \setminus v_j)^{\sigma'} & \text{if } \lceil v_i \sigma \rceil = \lceil v_j \sigma \rceil \text{ and } i \neq j \\ \text{XOR}(v_1^{\sigma'}, \dots, v_n^{\sigma'}) & \text{otherwise.} \end{cases}$$

$$\langle a, b \rangle^{\sigma'} = \langle a^{\sigma'}, b^{\sigma'} \rangle \quad \text{and similarly for } \{a\}_b^p \text{ and } \{a\}_b^s.$$

$$a^{\sigma'} = a \quad \text{if } a \in \text{Var} \cup \text{Var}' \cup \text{Atoms}.$$

where  $\text{XOR}(v_1, \dots, v_n \setminus v_i \setminus v_j)$  represents the XOR of all  $v_i$  for  $i \in \{1, \dots, n\} \setminus \{i, j\}$ . We can check that  $\lceil \lceil t \sigma'' \rceil \sigma' \rceil = \lceil t \sigma \rceil$  for all  $t$ , since by the above transformation we have  $\lceil \text{XOR}(v_1, \dots, v_n)^{\sigma'} \sigma' \rceil = \lceil \text{XOR}(v_1, \dots, v_n) \sigma \rceil$ . Moreover for all  $\text{XOR}(u_1, \dots, u_k)$  subterm of  $\lceil t \sigma'' \rceil$ , we have  $u_i \sigma'$  standard (since  $\lceil t \sigma'' \rceil$  is normalized and  $\sigma'(x_s)$  is standard, for all  $s \in F$ ), and it follows<sup>2</sup> that  $\lceil u_i \sigma \rceil \neq 0$ , and  $\lceil u_i \sigma \rceil \neq \lceil u_j \sigma \rceil$  for all  $i \neq j$ , by definition of  $\lceil t \sigma'' \rceil$ . By consequence  $\lceil t \sigma'' \rceil \sigma'$  is normalized and then :

$$\lceil \lceil t \sigma'' \rceil \sigma' \rceil = \lceil t \sigma \rceil \quad \text{for all terms } t.$$

We are now going to bound  $|S|$ . Given a set of normalized messages  $Z$ , let

$$\begin{aligned} V_Z &= \{x \in \mathcal{V} \mid \sigma(x) \text{ non-standard and } \sigma(x) \notin Z\}, \\ P_Z &= \{\lceil t \sigma'' \rceil \sigma' \mid t \in \mathcal{S}(P) \text{ and } \lceil t \sigma \rceil \notin Z\}. \end{aligned}$$

We note that  $Z \subseteq Z'$  implies  $V_{Z'} \subseteq V_Z$  and  $P_{Z'} \subseteq P_Z$ , and that  $V_S = \emptyset$ .

*Claim.*  $|S \cup P_S| \leq |V_\emptyset \cup P_\emptyset|$ .

<sup>2</sup> Remark that  $\sigma'(x) \neq 0$  for all  $x$

*Proof of the claim.* We construct a sequence of sets  $S = Z_1 \supset Z_2 \supset \dots \supset Z_n = \emptyset$  with  $Z_{i+1} = Z_i \setminus v_i$  where  $v_i \in Z_i$  is a maximal message in  $Z_i$  (w.r.t. the subterm ordering). Note that  $n-1$  is the cardinality of  $S$  and for every  $t \in Z_{i+1}$ ,  $v_i \notin \mathcal{F}(t)$ . For every  $i \in \{1, \dots, n\}$  we prove

$$|Z_i \cup V_{Z_i} \cup P_{Z_i}| \leq |Z_{i+1} \cup V_{Z_{i+1}} \cup P_{Z_{i+1}}|$$

which concludes the proof of the claim. At step  $i$ , either of two cases may arise when removing  $v = v_i \in Z_i$  from  $Z_i$ :

- There exists  $x \in \mathcal{V}$  with  $v = \sigma(x)$  non-standard. Then,

$$\begin{aligned} |Z_i \cup V_{Z_i} \cup P_{Z_i}| &\leq |Z_i \setminus v \cup \{x\} \cup \mathcal{F}(\sigma(x)) \cup V_{Z_i} \cup P_{Z_i}| \\ &\leq |Z_{i+1} \cup V_{Z_{i+1}} \cup P_{Z_{i+1}}| \end{aligned}$$

since  $x \notin Z_i \cup V_{Z_i}$ ,  $x \in V_{Z_{i+1}}$ , and  $\mathcal{F}(\sigma(x)) \subseteq Z_i \setminus v = Z_{i+1}$ .

- $v \in F$  and there exists  $t \in \mathcal{S}(P)$  such that  $t \sqsubseteq_\sigma v$ . Let  $t' = \ulcorner t \sigma \urcorner^{\sigma'}$ . We have  $t' \sigma' = \ulcorner t \sigma \urcorner = v$ . Then,

$$\begin{aligned} |Z_i \cup V_{Z_i} \cup P_{Z_i}| &\leq |Z_{i+1} \cup V_{Z_{i+1}} \cup \{t'\} \cup P_{Z_i}| \\ &\leq |Z_{i+1} \cup V_{Z_{i+1}} \cup P_{Z_{i+1}}| \end{aligned}$$

since  $\sigma'(y) \in Z_i \setminus v = Z_{i+1}$  for every  $y \in \mathcal{V}(t')$  and  $P_{Z_{i+1}} = P_{Z_i} \cup \{t'\}$ .

This proves the claim. Using the claim and

$$|P_\emptyset| = |\ulcorner \mathcal{S}(P) \mathcal{V} \urcorner| \leq |\mathcal{S}(P) \mathcal{V}'| \leq |\mathcal{S}(P)| + |\mathcal{V}'|$$

we obtain

$$\begin{aligned} |\{\sigma(x) \mid x \in \mathcal{V}\}| &\leq |S| \leq |S \cup P_S| \leq |V_\emptyset \cup P_\emptyset| \\ &\leq |\mathcal{V}| + |\mathcal{S}(P)| + |\mathcal{V}'| \leq 3 \cdot |P| \end{aligned}$$

□

From this theorem completeness of the procedure depicted in Figure 2 follows immediately.

## 7 Extending the Dolev-Yao Intruder by Different Oracle Rules

We extend the ability of the standard Dolev-Yao intruder beyond the perfect encryption hypothesis by considering two specific sets of oracle rules. The

first set are the XOR rules which allow the intruder to make use of the XOR operator. We then consider, what we call, prefix rules which allow the intruder to exploit certain properties of encryption based on block ciphers.

### 7.1 XOR Rules

The XOR rules allow the intruder to sum several messages with the XOR operator. The result of this sum is being normalized.

**Definition 21** We define  $L_o = L_{oc} \cup L_{od}$  to be the set of XOR rules where

- $L_{oc}$  is the set of rules of the form  $\{t_1, \dots, t_n\} \rightarrow \lceil \text{XOR}(t_1, \dots, t_n) \rceil$  with  $\{t_1, \dots, t_n\}$  a non-empty finite multiset of normalized messages such that  $\lceil \text{XOR}(t_1, \dots, t_n) \rceil$  is non-standard, and
- $L_{od}$  is the set of rules of the form  $\{t_1, \dots, t_n\} \rightarrow \lceil \text{XOR}(t_1, \dots, t_n) \rceil$  with  $\{t_1, \dots, t_n\}$  a non-empty finite multiset of normalized messages such that  $\lceil \text{XOR}(t_1, \dots, t_n) \rceil$  is standard.

We call the intruder using the rules  $L_o \cup L_c \cup L_d$  the XOR intruder.

Note that the rules in  $L_{od}$  are in fact decomposition guess rules since if  $\lceil \text{XOR}(t_1, \dots, t_n) \rceil$  is standard, it is a factor of some of the terms  $t_1, \dots, t_n$ . Note that we use that  $t_1, \dots, t_n$  are normalized. Also, the rules in  $L_{oc}$  are composition guess rules since proper subterms of  $\lceil \text{XOR}(t_1, \dots, t_n) \rceil$  are subterms of factors of this term, and thus, subterms of  $t_1, \dots, t_n$ . Again, we use that  $t_1, \dots, t_n$  are normalized.

We also note that the intruder is not more powerful if we allow him to derive non-normalized messages. More precisely, assume that  $L_e$  is the set of rules of the form  $\{t_1, \dots, t_n\} \rightarrow s$  with  $s =_{\text{XOR}} \text{XOR}(t_1, \dots, t_n)$  (not necessarily normalized). Let  $\text{forge}_e(E)$  denote the set of messages the intruder can derive from  $E$  with the rules  $L_e$ ,  $L_d$ , and  $L_c$ . Then, it easily follows by induction on the length of derivations:

**Proposition 22** For every message term  $t$  and set of messages  $E$  (both not necessarily normalized),  $t \in \text{forge}_e(E)$  implies  $\lceil t \rceil \in \text{forge}(\lceil E \rceil)$ .

Therefore, we can restrict the intruder to work only on normalized messages and to produce only normalized messages.

#### 7.1.1 Example

Before showing that the XOR rules are oracle rules, we illustrate that the XOR intruder can perform the attack informally described in Section 2.

We recall (see Section 3.2) that the protocol underlying the attack is formally described as follows: The initial intruder knowledge is  $\{0, I, ki, ki^{-1}, ka, kb\} = S_0$ . The protocol rules are

$$\begin{aligned}
(a, 1) : & \quad 0 \Rightarrow \{\langle na, a \rangle\}_{ki}^p \\
(a, 2) : & \quad \{\langle x_{\text{secret}}, \text{XOR}(na, I) \rangle\}_{ka}^p \Rightarrow \{x_{\text{secret}}\}_{ki}^p \\
(b, 1) : & \quad \{\langle x_{na}, a \rangle\}_{kb}^p \Rightarrow \{\langle \text{secret}, \text{XOR}(x_{na}, b) \rangle\}_{ka}^p
\end{aligned}$$

We have  $\mathcal{I} = \{(a, 1), (a, 2), (b, 1)\}$  and  $<_{\mathcal{I}} = \{((a, 1), (a, 2))\}$ .

When using a perfect encryption model, there is no attack on this instance of the protocol since the intruder is not able to forge  $\{\langle \text{secret}, \text{XOR}(na, I) \rangle\}_{ka}^p$  without the oracle rules. On the other hand, when using these rules,  $(\pi, \sigma)$  with the execution order  $\pi = \{(a, 1) \mapsto 0, (b, 1) \mapsto 1, (a, 2) \mapsto 2\}$  and the substitution  $\sigma$  with  $\sigma(x_{na}) = \text{XOR}(na, b, I)$  and  $\sigma(x_{\text{secret}}) = \text{secret}$  is an attack on this protocol. In fact, it is easy to check that the three following statements are true:

$$\{\langle \text{XOR}(na, b, I), a \rangle\}_{kb}^p \in \text{forge}(0, I, ki, ki^{-1}, ka, kb, \{\langle na, a \rangle\}_{ki}^p)$$

$$\begin{aligned}
\{\langle \text{secret}, \text{XOR}(na, I) \rangle\}_{ka}^p & \in \text{forge}(0, I, ki, ki^{-1}, ka, kb, \{\langle na, a \rangle\}_{ki}^p, \text{msg}) \\
& \text{with } \text{msg} = \ulcorner \{\langle \text{secret}, \text{XOR}(\text{XOR}(na, b, I), b) \rangle\}_{ka}^p \urcorner
\end{aligned}$$

$$\begin{aligned}
\text{secret} & \in \text{forge}(0, I, ki, ki^{-1}, ka, kb, \{\langle na, a \rangle\}_{ki}^p, \text{msg}, \{\langle \text{secret} \rangle\}_{ki}^p) \\
& \text{with } \text{msg} = \ulcorner \{\langle \text{secret}, \text{XOR}(\text{XOR}(na, b, I), b) \rangle\}_{ka}^p \urcorner
\end{aligned}$$

### 7.1.2 XOR rules are oracle rules.

We now show that the XOR rules form a set of oracle rules. We start to show Definition 7, (1). To do so, we first prove a sufficient condition for a derivation to be well-formed.

**Lemma 23** *Let  $D = E_0 \rightarrow_{L_1} \dots \rightarrow_{L_{n-1}} E_{n-1} \rightarrow_{L_n} E_n$  be a derivation with goal  $g$  such that:*

- (1) *For every  $j$  with  $E_{j-1} \rightarrow_{L_j} E_j$ ,  $t$  the  $j$ th step in  $D$  and  $L_j \in \mathcal{L}_d(t)$ , there exists  $t' \in E_{j-1}$  such that  $t$  is a subterm of  $t'$  and  $t' \in E_0$  or there exists  $i$  with  $i < j$  and  $L_i \in \mathcal{L}_d(t')$ .*
- (2) *For every  $i < n$  and  $t$  with  $L_i \in \mathcal{L}_c(t)$ , there exists  $j$  with  $i < j$  such that  $L_j$  is a  $t'$ -rule and  $t \in \mathcal{S}(E_0, t')$ .*

*Then,  $D$  is a well-formed derivation with goal  $g$ .*

*Proof.* From (1) it immediately follows by induction on  $i \in \{1, \dots, n\}$  that  $L_i \in \mathcal{L}_d(t)$  implies  $t \in \mathcal{S}(E_0)$  for every message  $t$ .

Using (2), we prove by induction on  $n - i$  that for all  $i \in \{1, \dots, n\}$ ,  $L_i \in \mathcal{L}_c(t)$  implies  $t \in \mathcal{S}(E_0, g)$ . If  $n - i = 0$ , then  $t = g$  and therefore  $t \in \mathcal{S}(E_0, g)$ . For the induction step, (2) implies that there exists  $j > i$  such that  $L_j$  is a  $t'$ -rule and  $t \in \mathcal{S}(E_0, t')$ . If  $L_j \in \mathcal{L}_d(t')$ , then  $t' \in \mathcal{S}(E_0)$  (see above). If  $L_j \in \mathcal{L}_c(t')$ , then by induction  $t' \in \mathcal{S}(E_0, g)$ , and hence,  $t \in \mathcal{S}(E_0, g)$ .  $\square$

Now, we can prove that XOR rules allow well-formed derivations.

**Proposition 24** *For every finite normalized set  $E$  of messages and normalized message  $g$ ,  $g \in \text{forge}(E)$  implies that there exists a well-formed derivation from  $E$  with goal  $g$ .*

*Proof.* Let  $E_0 = E$  and  $D = E_0 \rightarrow_{L_1} \dots \rightarrow_{L_n} E_n$  be a derivation of goal  $g$  of minimal length. We prove that  $D$  satisfies (1) and (2) in Lemma 23. We first show:

*Claim.* If  $F \rightarrow_{L_o(t)} F, t \rightarrow_{L_o(u)} F, t, u$ , then  $F \rightarrow_{L_o(u)} F, u$ .

*Proof of the claim.* By definition of XOR rules,  $u$  is a normalized XOR sum of elements in  $F, t$  and  $t$  is a normalized XOR sum of elements in  $F$ . Thus,  $u$  is an XOR sum of elements in  $F$ . Thus,  $F \rightarrow_{L_o(u)} F, u$ . This concludes the proof of the claim.

By the claim w.l.o.g. we may assume that in  $D$  the terms used on the left hand-side of an XOR rules are not generated by XOR rules. Formally (\*): For every  $i$  with  $L_i \in L_o(t)$  and  $L_i = F \rightarrow t$ , there does not exist  $j \in \{1, \dots, n\}$  such that  $L_j \in L_o(t')$  for some  $t' \in F$ .

Now, we prove (1) and (2) of Lemma 23:

- (1) If  $L_j \in L_d(s) \cap \mathcal{L}_d(t)$ , then  $L_i \notin L_{oc}(s)$ , for all  $i < j$ , since rules in  $L_{oc}$  do not create standard terms, and  $L_i \notin L_c(s)$ , for all  $i < j$ , by the minimality of  $D$  (otherwise  $L_j$  could be removed). Therefore, either  $s \in E$  or there exists  $i < j$  with  $L_i \in \mathcal{L}_d(s)$ .

If  $L_j \in L_{od}(t)$ , then  $t$  is standard and, by (\*) and the definition of  $L_{od}$ , there exists a non-standard term  $t'$  in  $E_{j-1}$  with  $t$  subterm of  $t'$  and such that  $L_{oc}(t') \notin D$ . If  $t' \in E$ , we are done. Otherwise, there exists  $j < i$  such that  $L_j$  is a  $t'$ -rule. If  $L_j \in \mathcal{L}_d(t')$ , again, we are done. Otherwise,  $L_j \in L_{oc}(t')$ , a contradiction to the choice of  $t'$ .

- (2) If  $L_i \in L_c(t)$  and  $i < n$ , then  $t$  is standard, and by minimality of  $D$ , there exists  $j > i$  such that  $t$  belongs to the left-hand side of  $L_j$ . By

definition of a derivation,  $L_j \notin L_d(t)$ . If  $L_j \in L_c(t')$ , then  $t \in \mathcal{S}(t')$ , and if  $L_j \in L_o(t')$ , then since  $t$  is standard, we have that  $t$  is a factor of  $t'$ , and thus,  $t \in \mathcal{S}(t')$ , or there exists  $t'' \in E_{j-1}$  non-standard with  $t \in \mathcal{S}(t'')$  ( $t$  is used to simplify  $t''$ ) such that, by (\*),  $t''$  was not generated by some rule in  $L_o$ . Since  $t''$  is non-standard it cannot be generated by some rule in  $L_c$  or  $L_d$ . Thus,  $t'' \in E_0$ .

If  $L_i \in L_{oc}(t)$  and  $i < n$ , then (\*) implies that  $t = g$  or there exists  $j > i$  such that  $L_i \in L_c(t')$  and  $t \in \mathcal{S}(t')$ . □

**Proposition 25** *The set  $L_o$  of XOR rules is a set of oracle rules.*

*Proof.* We check each condition in Definition 7:

- (1) The first point is a consequence of Proposition 24.
- (2) No term created with  $L_{oc}$  can be decomposed with  $L_d$ .
- (3) For  $F \rightarrow s \in L_{oc}(s)$ , every proper subterm of  $s$  is a subterm of  $F$  by the definition of  $L_{oc}$ .
- (4) For every non-atomic message  $u$  define  $\epsilon(u) = 0$ . Let  $u$  be a non-atomic message,  $F$  be a set of messages with  $0 \in F$  and  $t$  be a message such that  $F \setminus u \rightarrow_{\mathcal{L}_c(u)} F$  and  $F \rightarrow_{L_o(t)} F, t$ . Obviously,  $\epsilon(u) \in \text{forge}(F)$ . Let  $\delta = [u \leftarrow 0]$ . There are three cases:
  - (a) Either  $u = t$ . Then,  $t\delta = 0 \in \text{forge}(F\delta)$ .
  - (b) Or  $u \neq t$  and  $u$  is a pair or an encryption. Then, by the definition of XOR rules one easily verifies

$$\lceil F\delta \rceil \rightarrow_{L_o(t\delta)} \lceil F\delta, t\delta \rceil$$

- (c) Or  $u \neq t$  and  $u$  is non-standard. In particular,  $F \setminus u \rightarrow_{L_{oc}(u)} F$  and  $u = \text{XOR}(t_1, \dots, t_n)$  with  $t_1, \dots, t_n \in F \setminus u$ . Thus,  $F \setminus u \rightarrow_{L_o(t)} (F \setminus u), t$  since if  $u$  is needed in the construction of  $t$ , then the terms  $t_1, \dots, t_n$  can be used. Now, it easily follows that  $\lceil F\delta \rceil \rightarrow_{L_o(t\delta)} \lceil F\delta, t\delta \rceil$ . □

Also, we can show that XOR rules can be applied in polynomial time.

**Proposition 26** *Let  $L_o$  be the set of XOR rules. Then, the problem whether  $E \rightarrow t \in L_o(t)$ , for a given finite normalized set  $E$  of messages and a normalized message  $t$ , with set  $E, t$  represented as a DAG  $\mathcal{G}$ , can be decided in polynomial time with respect to  $|E, t|$ .*

*Proof.* Let  $B$  be the set of factors of terms in  $E$  and  $S$  be the factors of  $t$ , both can be represented as subsets of nodes of  $\mathcal{G}$ . Obviously,  $B$  and  $S$  can be obtained in polynomial time, and it can be decided in polynomial time whether  $S \subseteq B$ . If  $S \not\subseteq B$ , then  $t$  cannot be build from  $E$  using an XOR rule. Otherwise,  $S \subseteq B$ . We can represent  $t$  by  $\text{Factor}(t) \subseteq B$ . And this set

can be represented as a vector of length  $|B|$  with entries 0 and 1 where an entry indicates whether a message in  $B$  belongs to  $Factor(t)$  or not. This vector can be interpreted as an element of the vector space of dimension  $|B|$  over the field with two elements. In the same way the terms in  $E$  can be represented. Now, deciding  $E \rightarrow_{L_o(t)} E, t$  is equivalent to deciding whether the vector representing  $t$  can be represented as a linear combination of the vectors representing the messages in  $E$ . This can be done in polynomial time by gaussian elimination.  $\square$

As an immediate consequence of Theorem 8 we obtain that INSECURE with XOR rules is in NP. NP-hardness can be obtained as in [27]. Altogether this yields:

**Theorem 27** *INSECURE w.r.t. the XOR intruder is an NP-complete problem.*

Together with Proposition 26, Theorem 9 implies:

**Theorem 28** *For the XOR intruder, the problem DERIVE is in PTIME.*

In [12], this problem is called *ground reachability problem* and is only shown to be in NP.

## 7.2 Prefix Rules

As another instance of oracles rules, we consider what we call prefix rules. These rules allow the intruder to exploit certain properties of block encryption algorithms, based for example on cipher block chaining (CBC). Using Theorem 8, again we can show that INSECURE is NP-complete. The Section 7.2.1 provides an example that illustrates the intruder's additional power.

Throughout this section, we assume that terms do not contain the XOR operator and that the normalization function  $\lceil \cdot \rceil$  is the identity function. It is easy to verify that Theorem 8 also holds in this simplified setting.

### 7.2.1 Motivation

As an example, we use a variant of the Needham-Schroeder symmetric key authentication protocol [24], which is given as follows:

1.  $A \rightarrow S : A, B, N_A$
2.  $S \rightarrow A : \{N_A, B, K_{AB}, \{K_{AB}, A\}_{K_{BS}}^s\}_{K_{AS}}^s$
3.  $A \rightarrow B : \{K_{AB}, A\}_{K_{BS}}^s$

4.  $B \rightarrow A : \{N_B\}_{K_{AB}}^s$
5.  $A \rightarrow B : \{N_B - 1\}_{K_{AB}}^s$
6.  $B \rightarrow A : \{\text{secret}\}_{K_{AB}}$

This protocol is considered to be secure in [10]. However, a careful analysis of this protocol reveals a flaw in case encryption is carried out by cipher-block-chaining (CBC) and all atoms are of the size of a block [26]. The attack in [26] exploits that if a message  $m$  is encrypted using the CBC mode, then it is easy to obtain an encryption of the prefix of  $m$  under the same key even without knowledge of the encryption key. In other words, the intruder can perform the following intruder rule:

$$\{\langle M, M' \rangle\}_K \rightarrow \{M\}_K.$$

In the example above, the intruder can forge  $\{N_A, B\}_{K_{AS}}^s$  by applying such a rule on the second message of the protocol, i.e.,

$$\{\langle \langle N_A, B \rangle, K_{AB} \rangle, \{K_{AB}, A\}_{K_{BS}}^s \rangle\}_{K_{AS}}^s.$$

Then, the intruder can send this message to  $A$  in another session where  $B$  is the initiator of the protocol. In this second session (denoted by  $\cdot'$  below), the key  $N_A$  accepted by  $A$  is also known by the intruder, who can continue the communication with  $A$  and derive the **secret**. More precisely, the attack looks like this:

1.  $A \rightarrow S : A, B, N_A$
2.  $S \rightarrow A : \{N_A, B, K_{AB}, \{K_{AB}, A\}_{K_{BS}}^s\}_{K_{AS}}^s$
- 3'.  $I(B) \rightarrow A : \{N_A, B\}_{K_{AS}}^s$
- 4'.  $A \rightarrow I(B) : \{N'_A\}_{N_A}^s$
- 5'.  $I(B) \rightarrow A : \{N'_A\}_{N_A}^s$
- 6'.  $A \rightarrow I(B) : \{\text{secret}\}_{N_A}^s$

### 7.2.2 Prefix Rules are Oracle Rules

**Definition 29** We define  $L_o = L_{oc} \cup L_{od}$  to be the set of prefix rules where  $L_{od} = \emptyset$  and  $L_{oc}$  consists of intruder rules of the form

$$\{\langle \dots \langle \langle M, M_1 \rangle, M_2 \rangle, \dots \rangle, M_n \rangle\}_K^s \rightarrow_{L_{oc}} \{M\}_K^s$$

for any normalized messages  $K, M, M_1, \dots, M_n$ , ( $n \geq 1$ ). We call the intruder using the rule  $L_o \cup L_c \cup L_d$  prefix intruder.

We can prove that these *prefix* rules are oracle rules that can be checked in polynomial time and then conclude that INSECURE for an intruder equipped with prefix rules is NP-complete by Theorem 8.

We first show that prefix rules allow well-formed derivations and then verify the remaining oracle conditions.

**Proposition 30** *For all  $t \in \text{forge}(E)$ , there exists a well-formed derivation from  $E$  with goal  $t$ .*

*Proof.* Let  $E_0 = E$  and  $D = E_0 \rightarrow_{L_1} \dots \rightarrow_{L_n} E_n$  be a derivation of goal  $g$ . Let  $D'$  be a derivation obtained from  $D$  with the following transformation system where the rules are applied with priority order decreasing from 1 to 4.

- (1) If  $i < j$  with  $L_j \in L_{oc}(\{M\}_K^s)$  and  $L_i \in L_c(\langle \dots \langle M, M'_1 \rangle \dots, M'_p \rangle_K^s)$ , replace  $L_j$  by a sequence of  $L_d$  rules decomposing  $\langle \dots \langle M, M'_1 \rangle \dots, M'_p \rangle_K^s$  to  $M$  followed by  $L_c(\{M\}_K^s)$ . (Note that the number of  $L_{oc}$  rules strictly decreases.)
- (2) If  $i < j$  with  $L_i = \{M''\}_K^s \rightarrow \{M'\}_K^s \in L_{oc}$  and  $L_j = \{M'\}_K^s \rightarrow \{M\}_K^s \in L_{oc}$ , then replace  $L_j$  by the rule  $\{M''\}_K^s \rightarrow \{M\}_K^s$ . Note that the latter rule belongs to  $L_{oc}$ . The number of  $L_{oc}$  rules does not change but the size of the  $L_{oc}$  rule argument strictly increases. (This size is bounded by the biggest term in the derivation.)
- (3) If  $i < j$  with  $L_i = \{M'\}_K^s \rightarrow \{M\}_K^s \in L_{oc}$  and  $L_j \in L_d(\{M\}_K^s)$ , replace  $L_j$  by  $L_d(\{M'\}_K^s)$  followed by a sequence of  $L_d$  rules decomposing  $M'$  to  $M$ . (The  $L_{oc}$  rules do not change but the number of rules  $L_d(t)$  such that there exists  $L \in D$  with  $L \in L_{oc}(t)$  strictly decreases since, due to (2), there exists no  $L_{oc}(\{M'\}_K^s)$  rule in  $D$ .)
- (4) If there exists  $i < n$  such that  $L_i$  is a  $t$ -rule but, for all  $j > i$ ,  $L_j$  does not use  $t$ , then remove  $L_i$ . (This removes rules that produce messages not used in the derivation.)

Clearly, this transformation system terminates: This can easily be shown by defining a (well-founded) lexicographical ordering with the different components defined according to the remarks provided along with the transformations. Then, it is easy to observe that with every application of a transformation rule, the order of a derivation decreases w.r.t. the lexicographical ordering.

It is also clear that the derivation  $D'$  derived from  $D$  by exhaustively applying the transformation rules and eliminating redundant rules is in fact a derivation from  $E$  with goal  $g$ . We show that  $D' = E'_0 \rightarrow_{L'_1} \dots \rightarrow_{L'_m} E'_m$  is well-formed.

For any rule  $L'_i \in L_d(s)$  in  $D'$ ,  $s$  is neither obtained with  $L_c$  ( $L'_i$  would be useless) nor with  $L_{oc}$  due to transformation rule (3). Therefore, we have  $s \in E$  or there exists  $L'_j \in L_d(s)$  with  $j < i$  in  $D'$ . Iterating this argument, it follows that  $s$  is a subterm of  $E$ .

For  $\mathcal{L}_c$  rules, we will reason by induction on  $m-i$ , i.e., the induction hypothesis is that for any  $m \leq j > i$  and any message  $t'$  with  $L'_j \in \mathcal{L}_c(t')$  the condition on composition rules in well-formed derivation is satisfied. Now, assume that  $L'_i \in \mathcal{L}_c(t)$ . If  $m-i=0$ , then  $t=g$ , and therefore,  $t \in \mathcal{S}(E, g)$ . For the induction step, there exists a rule  $L'_j, j > i$ , in  $D'$  using  $t$ , by the transformation rule (4). If  $L'_i \in L_c(t)$ , it follows from the definition of derivations that  $L'_j \notin L_d(t)$ . If  $L'_i \in L_{oc}(t)$ , we also obtain  $L'_j \notin L_d(t)$ , by transformation rule (3). Thus, in any case,  $L'_j \notin L_d(t)$ . Using transformation rules (1) and (2), we can also conclude that  $L'_j \notin L_{oc}$ : While for the case that  $L'_i \in L_c(t)$ , this follows immediately by transformation (1), the case  $L'_i \in L_{oc}(t)$  is covered by transformation (2). Consequently,  $L'_j \in L_c(t')$ , and  $t$  is a subterm of  $t'$ . By the induction hypothesis and since  $j > i$ , we know that  $t' \in \mathcal{S}(g, E)$ , and thus,  $t \in \mathcal{S}(g, E)$ .  $\square$

We can now prove that these rules are oracle rules:

**Proposition 31** *The set  $L_o$  of prefix rules is a set of oracle rules.*

*Proof.* We check each point of the definition:

- (1) If  $t \in \text{forge}(E)$ , then there exists a well-formed derivation from  $E$  with goal  $t$ , thanks to Proposition 30.
- (2) If we have  $F \rightarrow_{L_{oc}(\{M\}_K^s)} F, \{M\}_K^s$  using  $\{M'\}_K^s$  and  $F, \{M\}_K^s \rightarrow_{L_d(\{M\}_K^s)} F, \{M\}_K^s, M$ , then as in transformation rule (3) in the proof of Proposition 30 one obtains a derivation from  $F$  with goal  $M$  without a rule in  $L_d(\{M\}_K)$ .
- (3) For any relation  $F \rightarrow_{L_{oc}(\{M\}_K^s)} F, \{M\}_K^s$  the proper subterms of  $\{M\}_K^s$  are the subterms of  $M$  and  $K$ , which are also subterms of  $F$ .
- (4) Let  $u$  be any non-atomic term. We choose  $\epsilon(\langle a, b \rangle) = a$  and  $\epsilon(u) = 0$  otherwise. Let  $F$  be a set of terms,  $0 \in F$ , and  $\{M\}_K^s$  a term such that  $F \setminus \{u\} \rightarrow_L F, u$ , with  $L \in \mathcal{L}_c(u)$ , and  $F \rightarrow_{L'} F, \{M\}_K^s$ , with  $L' \in L_{oc}(\{M\}_K^s)$ . Let  $\theta = [u \leftarrow \epsilon(u)]$ . We distinguish four cases:
  - (a) If  $u = \{M\}_K^s$ , then  $\{M\}_K^s \theta = 0 \in \text{forge}(F\theta)$ .
  - (b) Assume that  $L'$  uses  $u = \{\langle \dots \langle M, M'_1 \rangle \dots, M'_p \rangle\}_K^s$ . If  $L \in L_c(u)$ , it follows that  $M, K \in F \setminus u$ , and thus,  $\{M\}_K^s \theta \in \text{forge}(F\theta)$ . If  $L \in L_{oc}(u)$ , then  $\{\langle \dots \langle M, M'_1 \rangle \dots, M'_q \rangle\}_K^s \in F \setminus u$  for some  $q > p$  and messages  $M'_{p+1}, \dots, M'_q$ . Thus,  $\{M\}_K^s \theta \in \text{forge}(F\theta)$ .
  - (c) If  $L'$  uses  $\{\langle \dots \langle M, M'_1 \rangle \dots, M'_p \rangle\}_K^s$ ,  $1 \leq q \leq p$  and  $\langle \dots \langle M, M'_1 \rangle \dots, M'_q \rangle = u$ , then  $\{M\}_K^s \theta = \{M\}_K^s \in \text{forge}(\{t\}_K^s) \subseteq \text{forge}(F\theta)$  with  $t = \langle \dots \langle M, M'_1 \rangle \dots, M'_{q-1} \rangle, M'_{q+1} \rangle \dots, M'_p \rangle$ .
  - (d) Otherwise, if  $L'$  uses  $\{t\}_K^s$ , then  $\{M\}_K^s \theta \in \text{forge}(\{t\}_K^s \theta)$ .

$\square$

Obviously,  $E \rightarrow t \in L_o$  can be decided in polynomial time in  $|E, t|$ . Also, analogously to the proof in [27] one can show that INSECURE is NP-hard. Now, by Theorem 8, it follows:

**Theorem 32** INSECURE *w.r.t. the prefix intruder is an NP-complete problem.*

With Theorem 9 we obtain:

**Theorem 33** *For the prefix intruder, the problem DERIVE is in PTIME.*

## 8 Conclusion

Based on a general framework in which we equip the intruder with oracle rules, we have shown that when extending the standard Dolev-Yao intruder by i) rules for XORing messages or ii) rules which allow the intruder to make use of prefix properties of encryption algorithms the protocol insecurity problem for a finite number of sessions remains NP-complete. This is the first tight complexity bound given for the insecurity problem without the perfect encryption assumption. Here we have only considered insecurity as failure of secrecy. However, we believe that our result holds also for other properties that can be reduced to reachability problems in our model, such as authentication. Future work includes applying our approach to other kinds of intruder rules and algebraic laws such as those for RSA encryption and Diffie-Hellman exponentiation. First results have appeared in [7,8] is an important step in this direction. We have shown that security is decidable for a large class of protocols based on Diffie-Hellman key construction techniques, by reducing the problem to solving linear diophantine equations.

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